Analytic relations for partial alpha decay half-lives and barrier heights and positions

Guy Royer, C. Schreiber, H. Saulnier

ANALYTIC RELATIONS FOR PARTIAL ALPHA DECAY HALF-LIVES AND BARRIER HEIGHTS AND POSITIONS

G. ROYER, C. SCHREIBER, H. SAULNIER

Subatech, UMR: IN2P3/CNRS-Université-Ecole des Mines, 44307 Nantes Cedex 03, France
royer@subatech.in2p3.fr

From an adjustment on a recent selected data set of partial α-decay half-lives of 344 ground state to ground state transitions, analytic formulae are proposed depending on the angular momentum of the α particle. In particular, an expression allows to reproduce precisely the partial α-decay half-lives of even-even heavy nuclei and, then, to predict accurately the partial α-decay half-lives of other very heavy elements from the experimental or predicted Q_α. Simple expressions are also provided to calculate the potential barrier radius and height.

1. Introduction

In a previous study[1] formulae have been proposed to calculate the total α decay half-lives of 373 emitters having an α branching ratio close to one. The rms deviation between the theoretical and experimental values of $log_{10}T_\alpha(s)$ was respectively 0.285, 0.39, 0.36 and 0.35 for the 131 even-even, 106 even(Z)-odd(N), 86 odd-even and 50 odd-odd nuclei. The predicted power of these formulae has been verified recently[2] on new data and particularly for the heaviest elements.

In a recent paper[3], a carefully updated and selected partial α decay half-life data set of 344 ground-state-to-ground-state α transitions has been studied. The purpose of the present work is, firstly, to adjust the coefficients of the above-mentioned formulae[1] on this ground-state-to-ground-state decay data[3] in incorporating a l-dependence and, secondly, to provide simple expressions to determine the alpha-decay or capture barriers.

2. Alpha-Decay Half-Lives of Isotopes of Charge $Z = 117$

Very recently[4] the isotopes $^{293}117$ and $^{294}117$ were produced in fusion reactions between ^{48}Ca and ^{249}Bk. Two decay chains were identified. 5 events correspond to the isotope $^{293}117$ and 1 event to the isotope $^{294}117$. In the Table 1 the characteristics of the two cascades are given : the range of the experimental Q value and the experimental α-decay half-life and the values predicted using the above-mentioned

2   *G. Royer, C. Schreiber, H. Saulnier*

Table 1. Comparison between the experimental and calculated α-decay half-lives for the recent observed decay-chains originated from the isotopes A = 293 and A = 294 of the new element Z=117.

^A_Z	Q(MeV)	T_{exp}	T_{form}	^A_Z	Q(MeV)	T_{exp}	T_{form}
$^{293}_{117}$	11.1-11.26	10-25 ms	9.7-24 ms	$^{289}_{115}$	10.35-10.55	0.14-0.48 s	0.15-0.54 s
$^{285}_{113}$	9.65-9.85	3.7-10.5 s	3.1-12.0 s	$^{294}_{117}$	10.86-11.06	0.042-0.45 s	0.15-0.54 s
$^{290}_{115}$	10.05-10.13	0.016 s	1.18-323 s	$^{286}_{113}$	9.66-9.86	19.6 s	16.7-71.3 s
$^{282}_{111}$	9.03-9.23	0.51 s	314-1513 s	$^{278}_{109}$	9.5-9.88	7.6 s	0.48-7.1 s
$^{274}_{107}$	8.83-9.03	54 s	41-194 s				

formulae[1]. There is a very good agreement for the cascade starting from the ^{293}117 nucleus and for four nuclei of the other cascade. The disagreement is important for the ^{290}115 and ^{282}111 nuclei. In these two cases the experimental Q value is lower than expected.

3. Analytic L-Dependent Formulae for the Partial Alpha Decay Half-Lives

For the even-odd, odd-even and odd-odd nuclei the ground-state-to-ground-state transitions may occur for different spins and parities of the parent and daughter nuclei and, consequently, the α particle may take away an angular momentum l. According to the selection rules the minimal orbital angular momentum of the emitted α particle has been evaluated assuming that $l = 0$ for all even-even nuclei[3]. From these l values and for improving the accuracy of the preceding formulae an explicit dependence on l has been researched and the following empirical formulae are proposed. They lead respectively for the 136 even-even, 84 even-odd, 76 odd-even and 48 odd-odd nuclei to a rms deviation of 0.328, 0.5552, 0.6661 and 0.6807.

$$log_{10}\left[T\right] = -25.752 - 1.15055A^{\frac{1}{6}}\sqrt{Z} + \frac{1.5913Z}{\sqrt{Q}}, \tag{1}$$

$$log_{10}\left[T\right] = -27.750 - 1.1138A^{\frac{1}{6}}\sqrt{Z} + \frac{1.6378Z}{\sqrt{Q}} +$$
$$\frac{1.7383\ 10^{-6}ANZ[l(l+1)]^{\frac{1}{4}}}{Q} + 0.002457A[1-(-1)^l], \tag{2}$$

$$log_{10}\left[T\right] = -27.915 - 1.1292A^{\frac{1}{6}}\sqrt{Z} + \frac{1.6531Z}{\sqrt{Q}} +$$
$$\frac{8.9785\ 10^{-7}ANZ[l(l+1)]^{\frac{1}{4}}}{Q} + 0.002513A[1-(-1)^l], \tag{3}$$

$$log_{10}\left[T\right] = -26.448 - 1.1023A^{\frac{1}{6}}\sqrt{Z} + \frac{1.5967Z}{\sqrt{Q}} +$$
$$\frac{1.6961\ 10^{-6}ANZ[l(l+1)]^{\frac{1}{4}}}{Q} + 0.00101A[1-(-1)^l]. \tag{4}$$

Additionally for the 59 heavy ($N > 126$ and $Z > 82$) e-e nuclei of this data set the following formula

$$log_{10}[T] = -27.690 - 1.0441 A^{\frac{1}{6}}\sqrt{Z} + \frac{1.5702Z}{\sqrt{Q}} \tag{5}$$

leads to a very small rms deviation of 0.1867 while for the 77 remaining lighter e-e nuclei the expression

$$log_{10}[T] = -28.786 - 1.0329 A^{\frac{1}{6}}\sqrt{Z} + \frac{1.6127Z}{\sqrt{Q}} \tag{6}$$

leads to a rms deviation of only 0.2659.

The Q_α values[5], the experimental ground state to ground state α-decay half-lives and values evaluated from the formula (1) are given in Table (2). For most of the nuclei, the difference between the experimental and theoretical data is relatively weak.

4. Alpha Emission or Capture Barrier

The alpha decay barrier is strongly lowered by the proximity energy with regard to the pure Coulomb barrier and the top of the barrier moves to a more external position corresponding to two separated spheres maintained in unstable equilibrium by the balance between the repulsive Coulomb forces and the attractive nuclear proximity forces. The main part of the barrier corresponds to two-body shapes.

The following expression allows to determine rapidly and accurately the distance between the mass centers at the α barrier top. A and Z are the mass and charge of the mother nucleus.

$$R = 2.536 + 1.1157\,[4^{\frac{1}{3}} + (A - 4)^{\frac{1}{3}}] \quad fm. \tag{7}$$

The height of the barrier against α decay can be determined using:

$$E = -1.43 + \frac{e^2 \times 2 \times (Z - 2)}{2.536 + 1.1157[4^{\frac{1}{3}} + (A - 4)^{\frac{1}{3}}]} - Q \quad MeV, \tag{8}$$

from which the alpha-capture barrier height can be deduced in adding Q.

5. Conclusion

Empirical expressions depending on the angular momentum of the α particle for the even-odd, odd-even and odd-odd nuclei are proposed to determine $log_{10}T_{1/2}(s)$. The coefficients have been adjusted on a recent data set of partial α-decay half-lives of 344 ground state to ground state transitions. An accurate expression is provided to evaluate the partial α-decay half-lives of even-even heavy and superheavy elements from the experimental or predicted Q_α. Analytic expressions are given to evaluate rapidly the α-decay or capture barrier radius and height.

4   *G. Royer, C. Schreiber, H. Saulnier*

Table 2. Comparison between the decimal logarithms of the experimental and calculated with the formula (1) ground state to ground state α-decay half-lives (in s) for even-even nuclei.

^A_Z	Q	$logT_{exp}$	$logT_{form}$	^A_Z	Q	$logT_{exp}$	$logT_{form}$
$^{106}_{52}$	4.290	-4.15	-3.85	$^{112}_{54}$	3.33	2.53	2.78
$^{144}_{60}$	1.905	22.86	23.02	$^{148}_{62}$	1.986	23.34	23.42
$^{150}_{64}$	2.808	13.75	13.81	$^{150}_{66}$	4.351	3.08	3.05
$^{154}_{66}$	2.946	13.98	13.80	$^{154}_{68}$	4.28	4.68	4.59
$^{156}_{70}$	4.811	2.42	2.699	$^{156}_{72}$	6.028	-1.63	-1.74
$^{160}_{72}$	4.902	2.77	3.25	$^{174}_{72}$	2.497	22.8	23.7
$^{162}_{74}$	5.677	0.46	0.56	$^{166}_{74}$	4.856	4.74	4.48
$^{162}_{76}$	6.767	-2.73	-2.68	$^{168}_{76}$	5.818	0.62	0.83
$^{172}_{76}$	5.227	3.98	3.49	$^{186}_{76}$	2.823	22.8	22.3
$^{170}_{78}$	6.708	-1.85	-1.74	$^{176}_{78}$	5.885	1.22	1.36
$^{180}_{78}$	5.24	4.24	4.33	$^{190}_{78}$	3.251	19.31	18.72
$^{176}_{80}$	6.897	-1.69	-1.64	$^{182}_{80}$	5.997	1.86	1.74
$^{186}_{80}$	5.205	5.71	5.46	$^{186}_{82}$	6.47	0.68	0.66
$^{190}_{82}$	5.697	4.25	3.94	$^{194}_{82}$	4.738	9.99	9.13
$^{190}_{84}$	7.693	-2.59	-2.84	$^{194}_{84}$	6.987	-0.38	-0.55
$^{198}_{84}$	6.309	2.27	2.01	$^{202}_{84}$	5.701	5.13	4.69
$^{206}_{84}$	5.327	7.14	6.54	$^{212}_{84}$	8.954	-6.52	-6.83
$^{216}_{84}$	6.906	-0.84	-0.72	$^{198}_{86}$	7.349	-1.18	-1.03
$^{206}_{86}$	6.384	2.74	2.48	$^{210}_{86}$	6.159	3.95	3.38
$^{214}_{86}$	9.208	-6.57	-6.75	$^{218}_{86}$	7.263	-1.46	-1.15
$^{222}_{86}$	5.59	5.52	5.88	$^{210}_{88}$	7.152	0.57	0.30
$^{214}_{88}$	7.273	0.39	-0.22	$^{218}_{88}$	8.546	-4.59	-4.33
$^{222}_{88}$	6.679	1.59	1.88	$^{226}_{88}$	4.871	10.73	11.06
$^{218}_{90}$	9.849	-6.96	-6.89	$^{222}_{90}$	8.127	-2.69	-2.37
$^{226}_{90}$	6.45	3.39	3.70	$^{230}_{90}$	4.77	12.49	12.81
$^{226}_{92}$	7.701	-0.57	-0.23	$^{230}_{92}$	5.993	6.43	6.74
$^{234}_{92}$	4.858	13.04	13.28	$^{238}_{92}$	4.27	17.25	17.62
$^{234}_{94}$	6.31	5.89	6.11	$^{238}_{94}$	5.593	9.59	9.73
$^{242}_{94}$	4.985	13.18	13.40	$^{238}_{96}$	6.62	5.51	5.56
$^{242}_{96}$	6.216	7.28	7.38	$^{246}_{96}$	5.475	11.26	11.32
$^{240}_{98}$	7.719	2.03	1.99	$^{248}_{98}$	6.361	7.56	7.53
$^{252}_{98}$	6.217	8.01	8.17	$^{246}_{100}$	8.378	0.17	0.43
$^{250}_{100}$	7.557	3.38	3.26	$^{254}_{100}$	7.308	4.14	4.16
$^{252}_{102}$	8.55	0.74	0.55	$^{256}_{102}$	8.581	0.53	0.38

References

1. G. Royer, *J. Phys. G: Nucl. Part. Phys.* **26** (2000) 1149.
2. G. Royer and H. F. Zhang, *Phys. Rev. C* **77** (2008) 037602.
3. V. Yu. Denisov and A. A. Khudenko, *At. Data Nucl. Data Tables* **95** (2009) 815.
4. Yu. Ts. Oganessian *et al.*, *Phys. Rev. Lett.* **104** (2010) 142502.
5. G. Audi, A. H. Wapstra and C. Thibault, *Nucl. Phys. A* **729** (2003) 337.