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We list scalar leptoquarks that mediate proton decay via renormalizable couplings to the Standard Model fermions. We employ a general basis of baryon number violating operators to parameterize contributions of each leptoquark towards proton decay. This then sets the stage for investigation of bounds on the leptoquark couplings to fermions with respect to the most current Super Kamiokande results on proton stability. We quantify if, and when, it is necessary to have leptoquark masses close to a scale of grand unification in the realistic $SU(5)$ and flipped $SU(5)$ frameworks. The most conservative lower bounds on the leptoquark masses are then presented. We furthermore single out a leptoquark without phenomenologically dangerous tree-level exchanges that might explain discrepancy of the forward-backward asymmetries in $t\bar{t}$ production observed at Tevatron, if relatively light. The same state could also play significant role in explaining muon anomalous magnetic moment. We identify contributions of this leptoquark to dimension-six operators, mediated through a box diagram, and tree-level dimension-nine operators, that would destabilize proton if sizable leptoquark and diquark couplings were to be simultaneously present.

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I. PROTON DECAY LEPTOQUARKS

There has been a plethora of low-energy experiments capable of leptoquark discovery thus far. These have generated ever more stringent constraints on available parameter space for their existence. See, for example, [1–6] for some of the latest results. There also exists a large number of phenomenological studies of leptoquark signatures prompted primarily by various effects they could generate in flavor physics [7–11]. We are interested in a particular subset of scalar leptoquark states that are associated with proton decay. It is well-known that there exists only a small number of these states that can simultaneously violate baryon (B) and lepton (L) numbers [12–14]. The number of scalar leptoquarks that can mediate proton decay at the tree-level is even smaller [15]. Our aim is to present a comprehensive classification of leptoquarks and address a role these have in proton decay processes.

Scalar leptoquarks that mediate proton decay certainly represent qualitatively new physics. Although the relevant operators associated with exchange of these states can be studied from an effective theory point of view, we prefer to trace their origins to a particular unification scenario in order to expose their dependence on underlying couplings. In fact we will study these states in two different unification frameworks that correspond to the $SU(5)$ [16] and the flipped $SU(5)$ [17–19], i.e., $SU(5) \times U(1)$, embeddings of the mater fields. These two scenarios are general enough to cover other possible embedding schemes.

Let us start by spelling out qualitative differences between the scalar and vector, i.e., gauge boson, leptoquarks that mediate proton decay at the so-called dimension-six ($d = 6$) level. (The latter have been studied much more extensively in the literature. See, for example, [12–14, 17, 20–24].) Firstly, vector leptoquarks comprise twenty-four states whereas the scalar ones comprise eighteen (fifteen) states in case neutrinos are Dirac (Majorana) particles. Secondly, whereas $SU(5)$ contains only a half of all vector leptoquarks, the other half being in flipped $SU(5)$, one can already find all possible proton-decay mediating scalar leptoquarks in either $SU(5)$ or flipped $SU(5)$ framework. Hence, the scalar sector, although smaller, can potentially yield much richer structure with respect to the gauge one. Thirdly, the uncertainty in predictions for partial nucleon decay rates due to the gauge boson exchange resides entirely in a freedom to choose particular unitary rotations that need to be in agreement with observed mixing parameters in the fermionic sector as gauge bosons couple to matter with the gauge coupling strength. Scalar fields, on the other hand, couple to matter through Yukawa couplings. This brings additional uncertainties to potential predictions for relevant decay rates.

The leptoquark states that simultaneously violate B and L quantum numbers tend to mediate proton decay at tree-level and are therefore taken to be very massive. However, we have investigated an $SU(5)$ grand unified theory scenario [25] which resulted in a setup with a set of light leptoquarks. Namely, motivated by the need to explain anomalous events in $t\bar{t}$ production at Tevatron [26, 27], we have found that a light color triplet weak singlet scalar could contribute to $t\bar{t}$ production and explain the observed increase of the forward-backward asymmetry [28]. We have accordingly demonstrated that the unification of the fundamental interactions is possible if that set of light scalars is a part of the 45-dimensional representation [28].

In flavor physics, due to recent accurate measurements at Tevatron and LHCb, the presence of new physics (NP) in B systems seems rather unlikely. The muon anomalous magnetic moment, on the other hand, still leaves some room for NP contributions. The impact of potentially light leptoquark scalars, including the light color triplet weak singlet scalar, on the low energy and hadron collider phenomenology within that context has been investigated in Refs. [28–32].

The color triplet weak singlet scalar state we have singled out does not generate proton decay at the tree-level. However, one can still construct, as we show later, higher order loop diagrams that yield effective $d = 6$ and tree-level $d = 9$ operators which can destabilize proton. The natural question then is whether one can simultaneously address the $t\bar{t}$ asymmetry and the muon anomalous magnetic moment by using the very same leptoquark. We investigate this issue in detail in Sec. VI.

This paper is organized as follows. In Sections II and III we list all proton decay inducing leptoquarks in $SU(5)$ and flipped $SU(5)$ unification frameworks and specify their Yukawa couplings to the SM fermions. In Sec. IV we introduce the effective dimension-six operators for proton decay and calculate associated effective coefficients for each leptoquark state. Sec. V is devoted to a study of conservative lower bounds on the color triplet leptoquark mass within phenomenologically realistic $SU(5)$ and flipped $SU(5)$ scenarios. In Sec. VI we study leptoquarks that do not contribute to proton decay operators of dimension-6 at tree-level. We conclude in Sec. VII.

II. LEPTOQUARKS IN $SU(5)$

The scalars that couple to matter at tree-level reside in the 5-, 10-, 15-, 45- and 50-dimensional representations of $SU(5)$ because the SM matter fields comprise $\mathbf{10}_i$ and $\overline{\mathbf{5}}_j$, where $i, j = 1, 2, 3$ represent family indices. Namely, $\mathbf{10}_i = (\mathbf{1}, \mathbf{1}, 1)_i \oplus (\overline{\mathbf{3}}, \mathbf{1}, -2/3)_i \oplus (\mathbf{3}, \mathbf{2}, 1/6)_i = (e^C_i, u^C_i, Q_i)$ and $\overline{\mathbf{5}}_j = (\mathbf{1}, \mathbf{2}, -1/2)_j \oplus (\overline{\mathbf{3}}, \mathbf{1}, 1/3)_j = (L_j, d^C_j)$, where $Q_i = (u_i \quad d_i)^T$ and $L_j = (\nu_j \quad e_j)^T$ [16]. Possible contractions of the matter field representations hence read $\mathbf{10} \otimes \mathbf{10} = \overline{\mathbf{5}} \oplus \overline{\mathbf{45}} \oplus \overline{\mathbf{50}}$, $\mathbf{10} \otimes \overline{\mathbf{5}} = \mathbf{5} \oplus \mathbf{45}$ and $\overline{\mathbf{5}} \otimes \overline{\mathbf{5}} = \overline{\mathbf{10}} \oplus \overline{\mathbf{15}}$. Theory also allows for addition of right-handed neutrinos that can be introduced, for example, in the form of $SU(5)$ fermionic singlets $(\mathbf{1})$ without the need to enlarge the scalar sector. Note that one can also introduce additional non-trivial representations of matter to generate observed fermion mass parameters in the lepton [33] and quark [34] sectors. That, however, would not alter our operator analysis for large enough masses of extra matter fields.

Relevant decomposition of scalar representations to the SM gauge group, i.e., $SU(5) \to SU(3) \times SU(2) \times U(1)$, is given below [35]:

- $\mathbf{5} = (\mathbf{1}, \mathbf{2}, 1/2) \oplus (\mathbf{3}, \mathbf{1}, -1/3)$;
- $\mathbf{10} = (\mathbf{1}, \mathbf{1}, 1) \oplus (\overline{\mathbf{3}}, \mathbf{1}, -2/3) \oplus (\mathbf{3}, \mathbf{2}, 1/6)$;
- $\mathbf{15} = (\mathbf{1}, \mathbf{3}, 1) \oplus (\mathbf{3}, \mathbf{2}, 1/6) \oplus (\mathbf{6}, \mathbf{1}, -2/3)$;
- $\mathbf{45} = (\mathbf{8}, \mathbf{2}, 1/2) \oplus (\overline{\mathbf{6}}, \mathbf{1}, -1/3) \oplus (\mathbf{3}, \mathbf{3}, -1/3) \oplus (\overline{\mathbf{3}}, \mathbf{2}, -7/6) \oplus (\mathbf{3}, \mathbf{1}, -1/3) \oplus (\overline{\mathbf{3}}, \mathbf{1}, 4/3) \oplus (\mathbf{1}, \mathbf{2}, 1/2)$;
- $\mathbf{50} = (\mathbf{8}, \mathbf{2}, 1/2) \oplus (\mathbf{6}, \mathbf{1}, 4/3) \oplus (\overline{\mathbf{6}}, \mathbf{3}, -1/3) \oplus (\overline{\mathbf{3}}, \mathbf{2}, -7/6) \oplus (\mathbf{3}, \mathbf{1}, -1/3) \oplus (\mathbf{1}, \mathbf{1}, -2)$.

Only $\mathbf{5}$, $\mathbf{15}$ and $\mathbf{45}$ contain electrically neutral components and are thus capable of developing phenomenologically viable vacuum expectation values (VEVs). Contributions to the up-quark, down-quark and charged lepton masses can come from both $\mathbf{5}$ and $\mathbf{45}$ whereas Majorana (Dirac) masses for neutrinos can be generated by VEV of $\mathbf{15}$ ($\mathbf{5}$).

The scalar leptoquark states that violate both B and L quantum numbers are $(\mathbf{3}, \mathbf{1}, -1/3)$, $(\mathbf{3}, \mathbf{3}, -1/3)$ and $(\overline{\mathbf{3}}, \mathbf{1}, 4/3)$, if one *assumes* neutrinos to be Majorana particles. These states reside in $\mathbf{5}$, $\mathbf{45}$ and $\mathbf{50}$. However, if one allows for the possibility that neutrinos are Dirac particles there is another leptoquark—$(\overline{\mathbf{3}}, \mathbf{1}, -2/3)$—that is found in the $\mathbf{10}$ of $SU(5)$ that violates both B and L and could thus also destabilize proton. To that end we consider both the Majorana and Dirac neutrino cases to keep the analysis as general as possible. Altogether, there are eighteen (fifteen) scalar leptoquarks that could mediate proton decay in case neutrinos are Dirac (Majorana) particles. The leptoquarks in question are all triplets of color as they must contract with lepton and quark states into an $SU(3)$ singlet. Tables I, II, III and IV summarize couplings to the matter of relevant states that reside in 50-, 45-, 10- and 5-dimensional representations, respectively.

We observe that in the $SU(5)$ framework the primary obstacle to the proton stability seems to be the need to generate Yukawa couplings relevant for the charged lepton and down-quark masses. These receive equally important contributions from the $\mathbf{10}_i\overline{\mathbf{5}}_j\mathbf{5}^*$ and $\mathbf{10}_i\overline{\mathbf{5}}_j\mathbf{45}^*$ contractions [36]. It is clear from Tables II and IV that both of these, individually, generate potentially dangerous couplings. The up-quark Yukawa coupling generation, on the other hand, via the $\mathbf{10}_i\mathbf{10}_j\mathbf{45}$ operator seems not to pose any danger whatsoever as can be seen from the second column in Table II. However, that operator cannot generate viable masses for all up-quarks due to the antisymmetry of the corresponding mass matrix. The $\mathbf{10}_i\mathbf{10}_j\mathbf{5}$ contraction does provide

$SU(5)$	$Y^{10}_{ij}\mathbf{10}_i\mathbf{10}_j\mathbf{50}$
$(\mathbf{3},\mathbf{1},-1/3)$ $\equiv$ Δ	$12^{-1/2}\epsilon_{abc}[Y^{10}_{ij}+Y^{10}_{ji}]d^T_{a\,i}Cu_{b\,j}\Delta_c$ $3^{-1/2}[Y^{10}_{ij}+Y^{10}_{ji}]e^{C\,T}_iCu^C_{a\,j}\Delta_a$

TABLE I. Yukawa couplings of the B and L violating scalar in the 50-dimensional representation of $SU(5)$. $a,b,c = 1,2,3$ $(i,j = 1,2,3)$ are color (flavor) indices. Y^{10}_{ij} are Yukawa matrix elements associated with the relevant contraction in the group space of $SU(5)$.

$SU(5)$	$Y^{10}_{ij}\mathbf{10}_i\mathbf{10}_j\mathbf{45}$	$Y^{\overline{5}}_{ij}\mathbf{10}_i\overline{\mathbf{5}}_j\mathbf{45}^*$
$(\mathbf{3},\mathbf{1},-1/3)$ $\equiv$ Δ	$2^{1/2}[Y^{10}_{ij}-Y^{10}_{ji}]e^{C\,T}_iCu^C_{a\,j}\Delta_a$	$2^{-1}Y^{\overline{5}}_{ij}\epsilon_{abc}u^{C\,T}_{a\,i}Cd^C_{b\,j}\Delta^*_c$ $-2^{-1}Y^{\overline{5}}_{ij}u^T_{a\,i}Ce_j\Delta^*_a$ $2^{-1}Y^{\overline{5}}_{ij}d^T_{a\,i}C\nu_j\Delta^*_a$
$(\mathbf{3},\mathbf{3},-1/3)$ $\equiv$ $(\Delta^1,\Delta^2,\Delta^3)$	$2^{1/2}\epsilon_{abc}[Y^{10}_{ij}-Y^{10}_{ji}]d^T_{a\,i}Cd_{b\,j}\Delta^1_c$	$Y^{\overline{5}}_{ij}u^T_{a\,i}C\nu_j\Delta^{1*}_a$
	$-2\epsilon_{abc}[Y^{10}_{ij}-Y^{10}_{ji}]d^T_{a\,i}Cu_{b\,j}\Delta^2_c$	$2^{-1/2}Y^{\overline{5}}_{ij}u^T_{a\,i}Ce_j\Delta^{2*}_a$ $2^{-1/2}Y^{\overline{5}}_{ij}d^T_{a\,i}C\nu_j\Delta^{2*}_a$
	$-2^{1/2}\epsilon_{abc}[Y^{10}_{ij}-Y^{10}_{ji}]u^T_{a\,i}Cu_{b\,j}\Delta^3_c$	$-Y^{\overline{5}}_{ij}d^T_{a\,i}Ce_j\Delta^{3*}_a$
$(\overline{\mathbf{3}},\mathbf{1},4/3)$ $\equiv$ Δ	$2^{1/2}[Y^{10}_{ij}-Y^{10}_{ji}]\epsilon_{abc}u^{C\,T}_{i\,a}Cu^C_{b\,j}\Delta_c$	$-Y^{\overline{5}}_{ij}e^{C\,T}_iCd^C_{a\,j}\Delta^*_a$

TABLE II. Yukawa couplings of the B and L violating scalars in the 45-dimensional representation of $SU(5)$. $a,b,c = 1,2,3$ $(i,j = 1,2,3)$ are color (flavor) indices. Y^{10}_{ij} and $Y^{\overline{5}}_{ij}$ are Yukawa matrix elements.

viable up-quark masses but the price to pay is resurrection of the proton decay issue. To conclude, the only operator that can be considered innocuous in the Majorana neutrino case is the $\mathbf{10}_i\mathbf{10}_j\mathbf{45}$ contraction.

III. LEPTOQUARKS IN FLIPPED $SU(5)$

Another possibility to unify the SM matter into an $SU(5)$-based framework leads to the so-called flipped $SU(5)$ scenario [17–19]. A single family of matter fields in flipped $SU(5)$ can be seen as originating from a 16-dimensional representation of $SO(10)$. Actually, flipped $SU(5)$ is not necessarily completely embedded into $SO(10)$. Nevertheless, the generator of electric charge in flipped $SU(5)$ is given as a linear combination of a $U(1)$ generator that resides in $SU(5)$ and an extra $U(1)$ generator as if both of these originate from an $SO(10) \to SU(5)\times U(1)$ decomposition. This guarantees anomaly cancelation at the price of introducing one extra state per family, i.e., the right-handed neutrino ν^C. The transition between the $SU(5)$ and flipped

$SU(5)$	$Y^1_{ij}\mathbf{10}_i\mathbf{1}_j\mathbf{10}^*$	$Y^{\overline{5}}_{ij}\overline{\mathbf{5}}_i\overline{\mathbf{5}}_j\mathbf{10}$
$(\overline{\mathbf{3}},\mathbf{1},-2/3)$ $\equiv$ Δ	$Y^1_{ij}u^{C\,T}_{a\,i}C\nu^C_j\Delta^*_a$	$2^{-1/2}\epsilon_{abc}Y^{\overline{5}}_{ij}d^{C\,T}_{a\,i}Cd^C_{b\,j}\Delta_c$ $Y^{\overline{5}}=-Y^{\overline{5}\,T}$

TABLE III. Yukawa couplings of the B and L violating scalar in the 10-dimensional representation of $SU(5)$. $a,b,c=1,2,3$ ($i,j=1,2,3$) are color (flavor) indices. Y^1_{ij} and $Y^{\overline{5}}_{ij}$ are Yukawa matrix elements.

$SU(5)$	$Y^{10}_{ij}\mathbf{10}_i\mathbf{10}_j\mathbf{5}$	$Y^{\overline{5}}_{ij}\mathbf{10}_i\overline{\mathbf{5}}_j\mathbf{5}^*$	$Y^1_{ij}\overline{\mathbf{5}}_i\mathbf{1}_j\mathbf{5}$
$(\mathbf{3},\mathbf{1},-1/3)$ $\equiv$ Δ	$2\epsilon_{abc}[Y^{10}_{ij}+Y^{10}_{ji}]d^T_{a\,i}Cu_{b\,j}\Delta_c$ $-2[Y^{10}_{ij}+Y^{10}_{ji}]e^{C\,T}_iCu^C_{a\,j}\Delta_a$	$2^{-1/2}\epsilon_{abc}Y^{\overline{5}}_{ij}u^{C\,T}_{a\,i}Cd^C_{b\,j}\Delta^*_c$ $2^{-1/2}Y^{\overline{5}}_{ij}u^T_{a\,i}Ce_j\Delta^*_a$ $-2^{-1/2}Y^{\overline{5}}_{ij}d^T_{a\,i}C\nu_j\Delta^*_a$	$Y^1_{ij}d^{C\,T}_{a\,i}C\nu^C_j\Delta_a$

TABLE IV. Yukawa couplings of the B and L violating scalar in the 5-dimensional representation of $SU(5)$. $a,b,c=1,2,3$ ($i,j=1,2,3$) are color (flavor) indices. Y^{10}_{ij}, $Y^{\overline{5}}_{ij}$ and Y^1_{ij} are Yukawa matrix elements.

$SU(5)$ embeddings is then provided by $d^C\leftrightarrow u^C$, $e^C\leftrightarrow\nu^C$, $u\leftrightarrow d$ and $\nu\leftrightarrow e$ transformations. Flipped $SU(5)$ thus predicts existence of three right-handed neutrinos as these transform nontrivially under the underlying gauge symmetry.

The matter fields in flipped $SU(5)$ comprise $\mathbf{10}^{+1}_i$, $\overline{\mathbf{5}}^{-3}_i$ and $\mathbf{1}^{+5}_i$, where the superscripts correspond to the extra $U(1)$ charge assignment. To obtain the SM hypercharge Y one uses the relation $Y=(Y(U(1))-Y(U(1)_{SU(5)}))/5$, where $Y(U(1))$ and $Y(U(1)_{SU(5)})$ represent the quantum numbers of the extra $U(1)$ and the $U(1)$ in $SU(5)(\rightarrow SU(3)\times SU(2)\times U(1))$, respectively.

The scalar sector that can couple to matter directly is made out of $\mathbf{50}^{-2}$, $\mathbf{45}^{-2}$, $\mathbf{15}^{+6}$, $\mathbf{10}^{+6}$, $\mathbf{5}^{-2}$ and $\mathbf{1}^{-10}$. Representations that can generate contributions to the charged fermion masses and Dirac neutrino masses are $\mathbf{45}^{-2}$ and $\mathbf{5}^{-2}$, whereas Majorana mass for neutrinos can originate from interactions with $\mathbf{15}^{+6}$. Leptoquarks that violate B and L reside in $\mathbf{50}^{-2}$, $\mathbf{45}^{-2}$, $\mathbf{10}^{+6}$ and $\mathbf{5}^{-2}$ with relevant couplings to matter given in Tables V, VI, VIII and VII, respectively.

In flipped $SU(5)$ the main obstacle to matter stability is the the generation of the up-quark masses. Namely, these can be generated through $\mathbf{10}^{+1}_i\overline{\mathbf{5}}^{-3}_j\mathbf{5}^{*\,+2}$ and/or $\mathbf{10}^{+1}_i\overline{\mathbf{5}}^{-3}_j\mathbf{45}^{*\,+2}$ contractions. Both of these are dangerous as far as the proton decay is concerned as can be seen from Tables VI and VIII. All other contractions, in the Majorana neutrino case, are actually innocuous.

$SU(5)\times U(1)$	$Y^{10}_{ij}\mathbf{10}^{+1}_i\mathbf{10}^{+1}_j\mathbf{50}^{-2}$
$(\mathbf{3},\mathbf{1},-1/3)^{-2}$ $\equiv$ Δ	$12^{-1/2}\epsilon_{abc}[Y^{10}_{ij}+Y^{10}_{ji}]u^T_{a\,i}Cd_{b\,j}\Delta_c$ $3^{-1/2}[Y^{10}_{ij}+Y^{10}_{ji}]\nu^{C\,T}_i Cd^C_{a\,j}\Delta_a$

TABLE V. Yukawa couplings of the B and L violating scalar in 50-dimensional representation of $SU(5)$. $a,b,c=1,2,3$ ($i,j=1,2,3$) are color (flavor) indices. Y^{10}_{ij} are Yukawa matrix elements.

$SU(5)\times U(1)$	$Y^{10}_{ij}\mathbf{10}^{+1}_i\mathbf{10}^{+1}_j\mathbf{45}^{-2}$	$Y^{\overline{5}}_{ij}\mathbf{10}_i\overline{\mathbf{5}}^{-3}_j\mathbf{45}^{*\,+2}$
$(\mathbf{3},\mathbf{1},-1/3)^{-2}$ $\equiv$ Δ	$2^{1/2}[Y^{10}_{ij}-Y^{10}_{ji}]\nu^{C\,T}_i Cd^C_{a\,j}\Delta_a$	$2^{-1}Y^{\overline{5}}_{ij}\epsilon_{abc}d^{C\,T}_{a\,i}Cu^C_{b\,j}\Delta^*_c$ $-2^{-1}Y^{\overline{5}}_{ij}d^T_{a\,i}C\nu_j\Delta^*_a$ $2^{-1}Y^{\overline{5}}_{ij}u^T_{a\,i}Ce_j\Delta^*_a$
$(\mathbf{3},\mathbf{3},-1/3)^{-2}$ $\equiv$ $(\Delta^1,\Delta^2,\Delta^3)$	$2^{1/2}\epsilon_{abc}[Y^{10}_{ij}-Y^{10}_{ji}]u^T_{a\,i}Cu_{b\,j}\Delta^3_c$	$Y^{\overline{5}}_{ij}d^T_{a\,i}Ce_j\Delta^{3*}_a$
	$-2\epsilon_{abc}[Y^{10}_{ij}-Y^{10}_{ji}]u^T_{a\,i}Cd_{b\,j}\Delta^2_c$	$2^{-1/2}Y^{\overline{5}}_{ij}d^T_{a\,i}C\nu_j\Delta^{2*}_a$ $2^{-1/2}Y^{\overline{5}}_{ij}u^T_{a\,i}Ce_j\Delta^{2*}_a$
	$-2^{1/2}\epsilon_{abc}[Y^{10}_{ij}-Y^{10}_{ji}]d^T_{a\,i}Cd_{b\,j}\Delta^1_c$	$-Y^{\overline{5}}_{ij}u^T_{a\,i}C\nu_j\Delta^{1*}_a$
$(\overline{\mathbf{3}},\mathbf{1},4/3)^{-2}$ $\equiv$ Δ	$2^{1/2}[Y^{10}_{ij}-Y^{10}_{ji}]\epsilon_{abc}d^{C\,T}_{i\,a}Cd^C_{b\,j}\Delta_c$	$-Y^{\overline{5}}_{ij}\nu^{C\,T}_i Cu^C_{a\,j}\Delta^*_a$

TABLE VI. Yukawa couplings of the B and L violating scalars in 45-dimensional representation of $SU(5)$. $a,b,c=1,2,3$ ($i,j=1,2,3$) are color (flavor) indices. Y^{10}_{ij} and $Y^{\overline{5}}_{ij}$ are Yukawa matrix elements.

IV. PROTON DECAY

Let us discuss proton decay operators due to the scalar leptoquark exchange of the lowest possible dimension in detail. These are dimension-six operators made out of three quarks and a lepton that violate

$SU(5)\times U(1)$	$Y^{1}_{ij}\mathbf{10}^{+1}_i\mathbf{1}^{+5}_j\mathbf{10}^{*\,-6}$	$Y^{\overline{5}}_{ij}\overline{\mathbf{5}}_i^{-3}\overline{\mathbf{5}}_j^{-3}\mathbf{10}^{+6}$
$(\overline{\mathbf{3}},\mathbf{1},-2/3)^{+6}$ $\equiv$ Δ	$Y^1_{ij}d^{C\,T}_{a\,i}Ce^C_j\Delta^*_a$	$2^{-1/2}\epsilon_{abc}Y^{\overline{5}}_{ij}u^{C\,T}_{a\,i}Cu^C_{b\,j}\Delta_c$ $Y^{\overline{5}}=-Y^{\overline{5}\,T}$

TABLE VII. Yukawa couplings of the B and L violating scalar in 10-dimensional representation of $SU(5)$. $a,b,c=1,2,3$ ($i,j=1,2,3$) are color (flavor) indices. Y^1_{ij} and $Y^{\overline{5}}_{ij}$ are Yukawa matrix elements.

$SU(5)\times U(1)$	$Y^{10}_{ij}\mathbf{10}^{+1}_i\mathbf{10}^{+1}_j\mathbf{5}^{-2}$	$Y^{\overline{5}}_{ij}\mathbf{10}^{+1}_i\overline{\mathbf{5}}_j^{-3}\mathbf{5}^{*\,+2}$	$Y^1_{ij}\overline{\mathbf{5}}_i^{-3}\mathbf{1}^{+5}_j\mathbf{5}^{-2}$
$(\mathbf{3},\mathbf{1},-1/3)^{-2}$ $\equiv$ Δ	$-2\epsilon_{abc}[Y^{10}_{ij}+Y^{10}_{ji}]u^T_{a\,i}Cd_{b\,j}\Delta_c$ $-2[Y^{10}_{ij}+Y^{10}_{ji}]\nu^{C\,T}_iCd^C_{a\,j}\Delta_a$	$2^{-1/2}\epsilon_{abc}Y^{\overline{5}}_{ij}d^{C\,T}_{a\,i}Cu^C_{b\,j}\Delta^*_c$ $2^{-1/2}Y^{\overline{5}}_{ij}d^T_{a\,i}C\nu_j\Delta^*_a$ $-2^{-1/2}Y^{\overline{5}}_{ij}u^T_{a\,i}Ce_j\Delta^*_a$	$Y^1_{ij}u^{C\,T}_{a\,i}Ce^C_j\Delta_a$

TABLE VIII. Yukawa couplings of the B and L violating scalar in 5-dimensional representation of $SU(5)$. $a,b,c=1,2,3$ ($i,j=1,2,3$) are color (flavor) indices. Y^{10}_{ij}, $Y^{\overline{5}}_{ij}$ and Y^1_{ij} are Yukawa matrix elements.

B and L by 1 unit. They are summarized below

$$O_H(d_\alpha,e_\beta)=a(d_\alpha,e_\beta)\,u^T\,L\,C^{-1}\,d_\alpha\,u^T\,L\,C^{-1}e_\beta\,, \tag{1}$$

$$O_H(d_\alpha,e^C_\beta)=a(d_\alpha,e^C_\beta)\,u^T\,L\,C^{-1}\,d_\alpha\,e^{C\dagger}_\beta\,L\,C^{-1}u^{C*}\,, \tag{2}$$

$$O_H(d^C_\alpha,e_\beta)=a(d^C_\alpha,e_\beta)\,d^{C\dagger}_\alpha\,L\,C^{-1}\,u^{C*}\,u^T\,L\,C^{-1}e_\beta\,, \tag{3}$$

$$O_H(d^C_\alpha,e^C_\beta)=a(d^C_\alpha,e^C_\beta)\,d^{C\dagger}_\alpha\,L\,C^{-1}\,u^{C*}\,e^{C\dagger}_\beta\,L\,C^{-1}u^{C*}\,, \tag{4}$$

$$O_H(d_\alpha,d_\beta,\nu_i)=a(d_\alpha,d_\beta,\nu_i)\,u^T\,L\,C^{-1}\,d_\alpha\,d^T_\beta\,L\,C^{-1}\,\nu_i\,, \tag{5}$$

$$O_H(d_\alpha,d^C_\beta,\nu_i)=a(d_\alpha,d^C_\beta,\nu_i)\,d^{C\dagger}_\beta\,L\,C^{-1}\,u^{C*}\,d^T_\alpha\,L\,C^{-1}\,\nu_i\,, \tag{6}$$

$$O_H(d_\alpha,d^C_\beta,\nu^C_i)=a(d_\alpha,d^C_\beta,\nu^C_i)\,u^T\,L\,C^{-1}\,d_\alpha\,\nu^{C\dagger}_i\,L\,C^{-1}\,d^{C*}_\beta\,, \tag{7}$$

$$O_H(d^C_\alpha,d^C_\beta,\nu^C_i)=a(d^C_\alpha,d^C_\beta,\nu^C_i)\,d^{C\dagger}_\beta\,L\,C^{-1}\,u^{C*}\,\nu^{C\dagger}_i\,L\,C^{-1}\,d^{C*}_\alpha\,. \tag{8}$$

Here, $i(=1,2,3)$ and $\alpha,\beta(=1,2)$ are generation indices, where all operators that involve a neutrino are bound to have $\alpha+\beta<4$ due to kinematical constraints. $L(=(1-\gamma_5)/2)$ is the left projection operator. The $SU(3)$ color indices are not shown since the antisymmetric contraction $\epsilon_{abc}q_aq_bq_c$ is common to all the above listed operators. This notation has already been introduced in Ref. [24].

These operators allow one to write down explicitly $d=6$ proton decay contributions due to a particular leptoquark exchange [24]. To that end we first specify our convention for the redefinition of the fermion fields that yields the up-, down-quark and charged lepton mass matrices in physical basis: $M_{U,D,E}\to M^{\text{diag}}_{U,D,E}$. These are $U^T_CM_UU=M^{\text{diag}}_U$, $D^T_CM_DD=M^{\text{diag}}_D$, and $E^T_CM_EE=M^{\text{diag}}_E$. The quark mixing is

$U^\dagger D \equiv V_{UD} = K_1 V_{CKM} K_2$, where K_1 and K_2 are diagonal matrices containing three and two phases, respectively. In the neutrino sector we have $N_C^T M_N N = M_N^{\rm diag}$ ($N^T M_N N = M_N^{\rm diag}$) in the case of Dirac (Majorana) neutrinos. The leptonic mixing $E^\dagger N \equiv V_{EN} = K_3 V_{PMNS} K_4$ in case of Dirac neutrino, or $V_{EN} = K_3 V_{PMNS}$ in the Majorana case. K_3 is a diagonal matrix containing three phases whereas K_4 contains two phases. V_{CKM} (V_{PMNS}) is the Cabibbo-Kobayashi-Maskawa (Pontecorvo-Maki-Nakagawa-Sakata) mixing matrix.

A. Tree-level exchange ($d = 6$) operators in $SU(5)$

The only relevant coefficient for $\Delta \equiv (\mathbf{3}, \mathbf{1}, -1/3)$ from $\mathbf{50}$ is

$$a(d_\alpha, e^C_\beta) = \frac{1}{6m^2_\Delta}\,(U^T(Y^{10} + Y^{10\,T})D)_{1\alpha}\,(E^\dagger_C(Y^{10} + Y^{10\,T})^\dagger U^*_C)_{\beta 1}, \tag{9}$$

where m_Δ is a mass of leptoquark in question. (See Table I for details on notation for Yukawa couplings of the 50-dimensional representation to the matter.)

The relevant coefficients for $\Delta \equiv (\mathbf{3}, \mathbf{1}, -1/3)$ from $\mathbf{45}$ are

$$a(d^C_\alpha, e_\beta) = \frac{1}{4m^2_\Delta}\,(D^\dagger_C Y^{\overline{5}\,\dagger} U^*_C)_{\alpha 1}\,(U^T Y^{\overline{5}} E)_{1\beta}\,, \tag{10}$$

$$a(d^C_\alpha, e^C_\beta) = \frac{1}{\sqrt{2}m^2_\Delta}\,(D^\dagger_C Y^{\overline{5}\,\dagger} U^*_C)_{\alpha 1}\,(E^\dagger_C(Y^{10} - Y^{10\,T})^\dagger U^*_C)_{\beta 1}\,, \tag{11}$$

$$a(d_\alpha, d^C_\beta, \nu_i) = \frac{1}{4m^2_\Delta}\,(D^\dagger_C Y^{\overline{5}\,\dagger} U^*_C)_{\beta 1}\,(D^T Y^{\overline{5}} N)_{\alpha i}\,. \tag{12}$$

The relevant coefficients for $\Delta \equiv (\mathbf{3}, \mathbf{1}, -1/3)$ from $\mathbf{5}$ are

$$a(d_\alpha, e_\beta) = -\frac{\sqrt{2}}{m^2_\Delta}\,(U^T(Y^{10} + Y^{10\,T})D)_{1\alpha}\,(U^T Y^{\overline{5}} E)_{1\beta}\,, \tag{13}$$

$$a(d_\alpha, e^C_\beta) = -\frac{4}{m^2_\Delta}\,(U^T(Y^{10} + Y^{10\,T})D)_{1\alpha}\,(E^\dagger_C(Y^{10} + Y^{10\,T})^\dagger U^*_C)_{\beta 1},, \tag{14}$$

$$a(d^C_\alpha, e_\beta) = \frac{1}{2m^2_\Delta}\,(D^\dagger_C Y^{\overline{5}\,\dagger} U^*_C)_{\alpha 1}\,(U^T Y^{\overline{5}} E)_{1\beta},, \tag{15}$$

$$a(d^C_\alpha, e^C_\beta) = \frac{\sqrt{2}}{m^2_\Delta}\,(D^\dagger_C Y^{\overline{5}\,\dagger} U^*_C)_{\alpha 1}\,(E^\dagger_C(Y^{10} + Y^{10\,T})^\dagger U^*_C)_{\beta 1}\,, \tag{16}$$

$$a(d_\alpha, d_\beta, \nu_i) = \frac{\sqrt{2}}{m^2_\Delta}(U^T(Y^{10} + Y^{10\,T})D)_{1\alpha}\,(D^T Y^{\overline{5}} N)_{\beta i}\,, \tag{17}$$

$$a(d_\alpha, d^C_\beta, \nu_i) = -\frac{1}{2m^2_\Delta}\,(D^\dagger_C Y^{\overline{5}\,\dagger} U^*_C)_{\beta 1}\,(D^T Y^{\overline{5}} N)_{\alpha i}\,, \tag{18}$$

$$a(d_\alpha, d^C_\beta, \nu^C_i) = \frac{2}{m^2_\Delta}(U^T(Y^{10} + Y^{10\,T})D)_{1\alpha}\,(N^\dagger_C Y^{1\,\dagger} D^*_C)_{i\beta}\,, \tag{19}$$

$$a(d^C_\alpha, d^C_\beta, \nu^C_i) = -\frac{1}{\sqrt{2}m^2_\Delta}(D^\dagger_C Y^{\overline{5}\,\dagger} U^*_C)_{\beta 1}\,(N^\dagger_C Y^{1\,\dagger} D^*_C)_{i\alpha}\,. \tag{20}$$

The relevant coefficients for $\Delta^2 \in (\mathbf{3}, \mathbf{3}, -1/3)$ from $\mathbf{45}$ are

$$a(d_\alpha, e_\beta) = -\frac{\sqrt{2}}{M^2_{\Delta^2}} \, (U^T (Y^{10} - Y^{10\,T}) D)_{1\alpha} \, (U^T Y^{\overline{5}} E)_{1\beta} \,, \tag{21}$$

$$a(d_\alpha, d_\beta, \nu_i) = -\frac{\sqrt{2}}{M^2_{\Delta^2}} (U^T (Y^{10} - Y^{10\,T}) D)_{1\alpha} \, (D^T Y^{\overline{5}} N)_{\beta i} \,. \tag{22}$$

The relevant coefficient for $\Delta^1 \in (\mathbf{3}, \mathbf{3}, -1/3)$ from $\mathbf{45}$ is

$$a(d_\alpha, d_\beta, \nu_i) = \frac{2\sqrt{2}}{M^2_{\Delta^1}} (U^T Y^{\overline{5}} N)_{1i} \, (D^T (Y^{10} - Y^{10\,T}) D)_{\beta\alpha} \,, \tag{23}$$

where the extra factor of 2 comes from two terms in Fierz transformation

$$(\overline{s^C} L d)(\overline{\nu^C} L u) = -(\overline{u^C} L s)(\overline{\nu^C} L d) - (\overline{u^C} L d)(\overline{\nu^C} L s) \,. \tag{24}$$

The only relevant coefficient for $\Delta \equiv (\overline{\mathbf{3}}, \mathbf{1}, -2/3)$ from $\mathbf{10}$ is

$$a(d^C_\alpha, d^C_\beta, \nu^C_i) = -\frac{1}{\sqrt{2} m^2_\Delta} \, (D^\dagger_C (Y^{\overline{5}} - Y^{\overline{5}\,T})^\dagger D^*_C)_{\beta\alpha} \, (N^\dagger_C Y^{1\,\dagger} U^*_C)_{i1} \,. \tag{25}$$

Finally, $\Delta^3 \in (\mathbf{3}, \mathbf{3}, -1/3)$ and $(\overline{\mathbf{3}}, \mathbf{1}, 4/3)$, both from $\mathbf{45}$ of $SU(5)$, do not contribute to proton decay at tree-level. This is due to antisymmetry, in flavor space, of their couplings to the pair of up-quarks. Nevertheless, both states still induce proton decay through loops at an effective $d = 6$ level. We present systematic study of these contributions for the $(\overline{\mathbf{3}}, \mathbf{1}, 4/3)$ case in Section VI. There we also spell out contributions of the $(\overline{\mathbf{3}}, \mathbf{1}, 4/3)$ leptoquark to dimension-nine tree-level proton decay amplitudes. Equivalent contributions of $\Delta^3 \in (\mathbf{3}, \mathbf{3}, -1/3)$ are not pursued since the components Δ^1 and Δ^2 from the same state already contribute at leading order. In this manner, higher order contributions of $\Delta^{1,2,3}$ would only play a role of radiative corrections.

B. Tree-level exchange ($d = 6$) operators in flipped $SU(5)$

The only relevant coefficient for $\Delta \equiv (\mathbf{3}, \mathbf{1}, -1/3)^{-2}$ from $\mathbf{50}^{-2}$ is

$$a(d_\alpha, d^C_\beta, \nu^C_i) = \frac{1}{6 m^2_\Delta} \, (U^T (Y^{10} + Y^{10\,T}) D)_{1\alpha} \, (N^\dagger_C (Y^{10} + Y^{10\,T})^\dagger D^*_C)_{i\beta} \,. \tag{26}$$

The relevant coefficients for $\Delta \equiv (\mathbf{3}, \mathbf{1}, -1/3)^{-2}$ from $\mathbf{45}^{-2}$ are

$$a(d^C_\alpha, e_\beta) = \frac{1}{4 m^2_\Delta} \, (D^\dagger_C Y^{\overline{5}\,*} U^*_C)_{\alpha 1} \, (U^T Y^{\overline{5}} E)_{1\beta} \,, \tag{27}$$

$$a(d_\alpha, d^C_\beta, \nu_i) = -\frac{1}{4 m^2_\Delta} \, (D^\dagger_C Y^{\overline{5}\,*} U^*_C)_{\beta 1} \, (D^T Y^{\overline{5}} N)_{\alpha i} \,, \tag{28}$$

$$a(d^C_\alpha, d^C_\beta, \nu^C_i) = \frac{1}{\sqrt{2} m^2_\Delta} (D^*_C Y^{\overline{5}\,*} U^*_C)_{\beta 1} \, (N^\dagger_C (Y^{10} - Y^{10\,T})^\dagger D^*_C)_{i\alpha} \,. \tag{29}$$

The relevant coefficients for $\Delta \equiv (\mathbf{3}, \mathbf{1}, -1/3)^{-2}$ from $\mathbf{5}^{-2}$ are

$$a(d_\alpha, e_\beta) = \frac{\sqrt{2}}{m_\Delta^2}\, (U^T(Y^{10} + Y^{10\,T})D)_{1\alpha}\, (U^T Y^{\overline{5}} E)_{1\beta}\,, \tag{30}$$

$$a(d_\alpha, e_\beta^C) = \frac{2}{m_\Delta^2}\, (U^T(Y^{10} + Y^{10\,T})D)_{1\alpha}\, (E_C^\dagger Y^{1\,\dagger} U_C^*)_{\beta 1}\,, \tag{31}$$

$$a(d_\alpha^C, e_\beta) = \frac{1}{2m_\Delta^2}\, (D_C^\dagger Y^{\overline{5}\,*} U_C^*)_{\alpha 1}\, (U^T Y^{\overline{5}} E)_{1\beta}\,, \tag{32}$$

$$a(d_\alpha^C, e_\beta^C) = \frac{1}{\sqrt{2} m_\Delta^2}\, (D_C^\dagger Y^{\overline{5}\,*} U_C^*)_{\alpha 1}\, (E_C^\dagger Y^{1\,\dagger} U_C^*)_{\beta 1}\,, \tag{33}$$

$$a(d_\alpha, d_\beta, \nu_i) = \frac{-\sqrt{2}}{m_\Delta^2} (U^T(Y^{10} + Y^{10\,T})D)_{1\alpha}\, (N^T Y^{\overline{5}\,T} D)_{i\beta}\,, \tag{34}$$

$$a(d_\alpha, d_\beta^C, \nu_i) = \frac{-1}{2m_\Delta^2}\, (D_C^\dagger Y^{\overline{5}\,*} U_C^*)_{\beta 1}\, (D^T Y^{\overline{5}} N)_{\alpha i}\,, \tag{35}$$

$$a(d_\alpha, d_\beta^C, \nu_i^C) = \frac{-4}{m_\Delta^2}\, (U^T(Y^{10} + Y^{10\,T})D)_{1\alpha}\, (N_C^\dagger (Y^{10} + Y^{10\,T})^\dagger D_C^*)_{i\beta}\,, \tag{36}$$

$$a(d_\alpha^C, d_\beta^C, \nu_i^C) = -\frac{\sqrt{2}}{m_\Delta^2} (D_C^\dagger Y^{\overline{5}\,*} U_C^*)_{1\beta}\, (N_C^\dagger (Y^{10} + Y^{10\,T})^\dagger D_C^*)_{i\alpha}\,. \tag{37}$$

The relevant coefficients for $\Delta^2 \in (\mathbf{3}, \mathbf{3}, -1/3)^{-2}$ from $\mathbf{45}^{-2}$ are

$$a(d_\alpha, e_\beta) = -\frac{\sqrt{2}}{M_{\Delta^2}^2}\, (U^T(Y^{10} - Y^{10\,T})D)_{1\alpha}\, (U^T Y^{\overline{5}} E)_{1\beta}\,, \tag{38}$$

$$a(d_\alpha, d_\beta, \nu_i) = -\frac{\sqrt{2}}{M_{\Delta^2}^2} (U^T(Y^{10} - Y^{10\,T})D)_{1\alpha}\, (D^T Y^{\overline{5}} N)_{\beta i}\,. \tag{39}$$

The relevant coefficient for $\Delta^1 \in (\mathbf{3}, \mathbf{3}, -1/3)^{-2}$ from $\mathbf{45}^{-2}$ is

$$a(d_\alpha, d_\beta, \nu_i) = \frac{2\sqrt{2}}{M_{\Delta^1}^2} (U^T Y^{\overline{5}} N)_{1i}\, (D^T(Y^{10} - Y^{10\,T})D)_{\beta\alpha}\,, \tag{40}$$

where the extra factor of 2 comes from Fierz transformation (24).

The only relevant coefficient for $\Delta \equiv (\mathbf{3}, \mathbf{1}, 4/3)^{-2}$ from $\mathbf{45}^{-2}$ is

$$a(d_\alpha^C, d_\beta^C, \nu_i^C) = -\frac{2\sqrt{2}}{M_{\Delta^1}^2} (U_C^\dagger Y^{\overline{5}\,\dagger} N_C^*)_{i1}\, (D_C^\dagger (Y^{10} - Y^{10\,T})^\dagger D_C^*)_{\alpha\beta}\,, \tag{41}$$

where the extra factor of 2 again comes from Fierz transformation.

The relevant coefficients for $(\overline{\mathbf{3}}, \mathbf{1}, -2/3)^{+6} \in \mathbf{10}^{+6}$ and $\Delta^3 \in (\mathbf{3}, \mathbf{3}, -1/3)^{-2} \in \mathbf{45}^{-2}$ are not present at the leading order due to antisymmetry of the couplings to the up-quark pair. The higher order contributions of the former state are discussed in Section VI.

V. LEADING ORDER CONTRIBUTIONS

Scalar fields couple to matter through Yukawa couplings. This introduces uncertainties to predictions related to any processes that involve scalar exchange. It is thus natural to ask if, and when, it is necessary to have leptoquark masses close to a scale of grand unification.

A. Color triplets of $SU(5)$

The lack of predictivity in the scalar sector can best be demonstrated if we revisit the case of the triplet scalar in the 5-dimensional representation of $SU(5)$. Recall, it is this scalar that has led to the so-called doublet-triplet splitting problem within the context of the Georgi-Glashow $SU(5)$ model [16].

Operators associated with the triplet exchange are given in Eqs. (13) through (20). One can see that all these operators, in the Majorana neutrino case, can be suppressed if $(U^T(Y^{10}+Y^{10\,T})D)_{1\alpha}=0$ and $(D_C^\dagger Y^{\overline{5}\,\dagger} U_C^*)_{\alpha 1}=0$ for $\alpha=1,2$. Here we assume not only that the triplet is the only relevant scalar state that mediates proton decay but that the entries of U, U_C, D, D_C, Y^{10} and $Y^{\overline{5}}$ are all free parameters. The first set of conditions can be insured if, for example, $Y^{10}=-Y^{10\,T}$. This solution has already been pointed out in Ref. [24]. The second set of conditions can also be easily satisfied although what we find defers from what has been presented in [24].

Clearly, if Yukawa sector relevant for proton decay through scalar exchange is not related to the origin of fermion masses one cannot make any firm predictions. Things, however, might change in models where that connection is strong. Let us thus analyze predictions of the simplest of all possible renormalizable models based on the $SU(5)$ gauge symmetry. We want to find what the current experimental bounds on $p\to e^+\pi^0$ and $p\to\mu^+\pi^0$ partial lifetimes imply for the masses of color triplets if the theory is to be viable with regard to the fermion mass generation. We present analysis for these particular decays because they are well-constrained experimentally and they turn out to be the least model dependent.

The starting point for our analysis are the following relations for the decay widths [24] for the $p\to e_\delta^+\pi^0$ ($\delta=1,2$ corresponding to e, μ) channels

$$\Gamma(p\to e_\delta^+\pi^0)=\frac{m_p}{64\pi f_\pi^2}\left(|\alpha a(d_1,e_\delta)+\beta a(d_1^C,e_\delta)|^2+|\alpha a(d_1,e_\delta^C)+\beta a(d_1^C,e_\delta^C)|^2\right)(1+D+F)^2, \quad (42)$$

where α and β are the so-called nucleon matrix elements. $F+D$ and $F-D$ combinations are extracted from the nucleon axial charge and form factors in semileptonic hyperon decays, respectively [37, 38]. We take in what follows $\alpha=-\beta=-0.0112(25)\,\mathrm{GeV}^3$. (See Section VI for more details on α and β.) We further take $f_\pi=130\,\mathrm{MeV}$, $m_p=938.3\,\mathrm{MeV}$, $D=0.80(1)$ and $F=0.47(1)$ [38].

We demand that the theory is renormalizable and thus neglect possibility that higher-dimensional terms contribute to (super)potential at any level. We furthermore take the simplest possibility for the generation of phenomenologically viable fermion masses. Namely, we demand that both **5** and **45** of Higgs contribute to the down-quark and charged lepton masses [36]. We further take all mass matrices to be symmetric, i.e.,

$M_{U,D,E} = M_{U,D,E}^T$. With these assumptions we obtain

$$\Gamma(p \to e_\delta^+ \pi^0) = \frac{m_p}{64\pi f_\pi^2} \frac{\alpha^2}{v_5^4 m_\Delta^4} \left|(V_{UD})_{11}[m_u + \frac{3}{4}m_d] + \frac{1}{4}(V_{UD}^\dagger U_2^* M_E^{diag} U_2^\dagger)_{11}\right|^2$$
$$\left(\left|\frac{3}{2}(V_{UD}^* M_D^{diag} V_{UD}^\dagger U_2^*)_{1\delta} + \frac{1}{2}(U_2 M_E^{diag})_{1\delta}\right|^2 + 4|m_u(U_2)_{1\delta}|^2\right)(1+D+F)^2,$$

where $U_2 = U^T E^*$ while v_5 represents the VEV of the 5-dimensional representation. U_2 entries and v_5 are primary sources of uncertainty. Our normalization is such that $|v_5|^2/2 + 12|v_{45}|^2 = v^2$, where $v(= 246\,\mathrm{GeV})$ stands for the electroweak VEV. v_{45} is the VEV in the 45-dimensional representation. A connection between Yukawa couplings and charged fermion mass matrices is spelled out elsewhere [31].

We see that the uncertainty in predicting partial decay rates persists even in this realistic scenario. One can suppress $\Gamma(p \to e_\delta^+ \pi^0)$ with regard to U_2 numerically to generate the least conservative lower bound on the mass of the scalar triplet in the 5-dimensional representation of $SU(5)$. This, however, should be simultaneously done with all other partial decay rates to generate a consistent solution. This is beyond the scope of this study. The important point is that even in the case of symmetric Yukawa couplings one cannot test the theory if the scalar triplet exchange dominates. This is in stark contrast to what one obtains for the gauge $d = 6$ contributions in the $SU(5)$ framework [20].

We can maximize $\Gamma(p \to e_\delta^+ \pi^0)$ with regard to U_2 to obtain the most conservative bound, from the model building point of view, on m_Δ. In fact, one can actually reach the maximal value for $\Gamma(p \to e_\delta^+ \pi^0)$ by simply taking

$$U_2 = U^T E^* = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \tag{43}$$

With this ansatz we obtain the following simplified expressions:

$$\Gamma(p \to e^+ \pi^0) \approx \frac{3^2 m_p \alpha^2}{2^{12}\pi f_\pi^2 v_5^4 m_\Delta^4} \left|(V_{UD})_{13}\right|^2 m_b^2 m_\tau^2 (1+D+F)^2\,, \tag{44}$$

$$\Gamma(p \to \mu^+ \pi^0) \approx \frac{3^2 m_p \alpha^2}{2^{12}\pi f_\pi^2 v_5^4 m_\Delta^4} \left|(V_{UD})_{12}\right|^2 m_s^2 m_\tau^2 (1+D+F)^2\,. \tag{45}$$

These expressions when combined with experimental input yield comparable bounds on the mass of the leptoquark in question. We use the following limits [39]

$$\tau(p \to \pi^0 e^+) > 1.3 \times 10^{34}\,\mathrm{yrs} \quad @\,90\%\,\mathrm{CL} \tag{46a}$$

$$\tau(p \to \pi^0 \mu^+) > 1.1 \times 10^{34}\,\mathrm{yrs} \quad @\,90\%\,\mathrm{CL} \tag{46b}$$

The $p \to \mu^+\pi^0$ channel gives slightly more stringent bound that reads

$$m_\Delta > 3.6 \times 10^{12} \left(\frac{\alpha}{0.0112\,\mathrm{GeV}^3}\right)^{1/2} \left(\frac{100\,\mathrm{GeV}}{v_5}\right)\,\mathrm{GeV}. \tag{47}$$

Here we take values of quark and lepton masses at M_Z, as given in [40] , and neglect any running of the relevant coefficients, for simplicity. These effects can be accounted for in a straightforward manner.

In the previous analysis it was assumed that contributions to proton decay of the triplet in the 5-dimensional representation dominate over contributions of triplets in the 45-dimensional representation. Let us now see what happens if that is not the case. Note that our assumption that the mass matrices are symmetric implies that the only contribution to $p \to e_\delta^+\pi^0$ ($\delta = 1, 2$) channels one needs to consider with regard to the $\mathbf{45}$ triplet exchanges originates from the $(\mathbf{3}, \mathbf{1}, -1/3)$ state. We accordingly find

$$\Gamma(p \to e_\delta^+\pi^0) = \frac{m_p\alpha^2}{2^{18}\pi f_\pi^2 v_{45}^4 m_\Delta^4} \left|(V_{UD}^\dagger U_2^* M_E^{diag} U_2^\dagger)_{11} - (V_{UD})_{11} m_u\right|^2$$
$$\left|(U_2 M_E^{diag} - V_{UD}^* M_D^{diag} V_{UD}^\dagger U_2^*)_{1\delta}\right|^2 (1 + D + F)^2\,,$$

where v_{45} represents the VEV of the 45-dimensional representation. With the ansatz given in Eq. (43) we obtain the following expressions

$$\Gamma(p \to e^+\pi^0) \approx \frac{m_p\alpha^2}{2^{18}\pi f_\pi^2 v_{45}^4 m_\Delta^4} \left|(V_{UD})_{13}\right|^2 m_b^2 m_\tau^2 (1 + D + F)^2\,, \tag{48}$$

$$\Gamma(p \to \mu^+\pi^0) \approx \frac{m_p\alpha^2}{2^{18}\pi f_\pi^2 v_{45}^4 m_\Delta^4} \left|(V_{UD})_{12}\right|^2 m_s^2 m_\tau^2 (1 + D + F)^2\,. \tag{49}$$

These can be compared with the results for the exchange of the 5-dimensional triplet given in Eqs. (44) and (45) to obtain

$$\left(\frac{\Gamma(p \to e_\delta^+\pi^0)^5}{\Gamma(p \to e_\delta^+\pi^0)^{45}}\right)_{max} = 576 \left(\frac{v_{45}}{v_5}\right)^4, \qquad \delta = 1, 2. \tag{50}$$

We see that the 5-dimensional triplet exchange dominates over the 45-dimensional triplet for moderate values of v_{45}.

To summarize, if one is to maximize contributions from the triplets in the 5- and 45-dimensional representations towards proton decay within renormalizable $SU(5)$ framework with symmetric mass matrices the current, most conservative, bound on the triplet mass scale is given in Eq. (47) (Eq. (50)) if the color triplet in $\mathbf{5}$ ($\mathbf{45}$) dominates. In other words, any such $SU(5)$ scenario where the triplet scalar mass exceeds this bound is certainly safe with regard to the proton decay constraints. If triplet is to be lighter than that, one needs to explicitly check if particular implementation of Yukawa couplings allows for such scenario. Also, any mixing between the triplets can be accounted for by simple rescaling of relevant operators.

B. Color triplet in flipped $SU(5)$

Flipped $SU(5)$ is well-known for the so-called missing partner mechanism that naturally addresses scalar mediated proton decay by making the relevant triplet scalar in the 5-dimensional representation heavy. Be that as it may, it is easy to see that the operators associated with the triplet exchange in flipped $SU(5)$ given in Eqs. (30) through (35), in the Majorana neutrino case, can be suppressed with ease if $(U^T(Y^{10} + Y^{10\,T})D)_{1\alpha} = 0$ and $(D_C^\dagger Y^{\bar{5}*} U_C^*)_{\alpha 1} = 0$ for $\alpha = 1, 2$. (In flipped $SU(5)$ the Dirac mass matrix for neutrinos is proportional to the up-quark mass matrix. This implies that flipped $SU(5)$ predicts Majorana nature of neutrinos.)

In the minimal realistic version of flipped $SU(5)$ it is sufficient to have only one scalar 5-dimensional representation present to generate realistic charged fermion masses. Let us then analyze the prediction of such scenario. To be able to compare these results with what we obtain in the case of ordinary $SU(5)$ we again take $M_{U,D,E} = M_{U,D,E}^T$. We get the following result for the $p \to e_\delta^+ \pi^0$ ($\delta = 1, 2$) partial decay widths

$$\Gamma(p \to e_\delta^+ \pi^0) = \frac{m_p}{64\pi f^2} \frac{\alpha^2}{v_5^4 m_\Delta^4} \left|(V_{UD})_{11}(m_d - m_u)\right|^2 4(m_u^2 + m_e^2)|(U_2)_{1\delta}|^2(1 + D + F)^2, \tag{51}$$

where $U_2 = U^T E^*$ and $v_5 (= \sqrt{2}\, 246\,\text{GeV})$ represents the VEV of the 5-dimensional representation. Interestingly enough, if U_2 takes the form given in Eq. (43) it would yield suppressed partial decay widths for $p \to e_\delta^+ \pi^0$, $\delta = 1, 2$. In other words, the setup that enhances certain partial proton decay rates in $SU(5)$ would suppress corresponding rates in flipped $SU(5)$ framework.

If we want to be conservative with regard to the limit on m_Δ it is sufficient to maximize relevant decay widths. This can be done by taking $(U_2)_{1\delta} = 1$ for $p \to e_\delta^+ \pi^0$ ($\delta = 1, 2$) to obtain

$$m_\Delta > 3.6 \times 10^{10} \left(\frac{\alpha}{0.0112\,\text{GeV}^3}\right)^{1/2} \text{GeV}. \tag{52}$$

This limit comes out to be significantly weaker with respect to the corresponding limit we presented in the case of $SU(5)$.

VI. HIGHER ORDER CONTRIBUTIONS

In the $SU(5)$ framework the states $(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$ and $\Delta^3 \in (\mathbf{3}, \mathbf{3}, -1/3)$ violate B and L and do not contribute to dimension-six proton decay operators at tree-level. Antisymmetry of their Yukawa couplings to two up quarks only allows for dimension-six operators involving c or t quarks that produce B number violation in charm or top decays [41], but these operators do not affect the proton stability due to large masses of c and t quarks. However, an additional W boson exchange opens decay channels with final states that are kinematically accessible to proton decay.

A. Box mediated dimension-six operator from $(\bar{\mathbf{3}}, \mathbf{1}, 4/3) \in 45$

One possibility is to make a box diagram with a single W exchange leading to the $d = 6$ operator, as shown on Fig. 1. In the literature, proton decay mediation involving W boson exchanges were considered

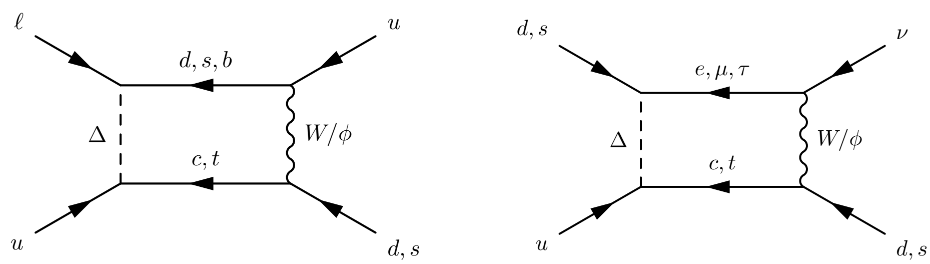

FIG. 1. Box diagrams with $(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$ state that generate $d = 6$ operators of flavor $uud\ell$ and $udd\nu$.

in [41–43]. We calculate the box diagram in the approximation where we neglect external momenta, however, we keep both virtual fermions massive since the right-handed Δ interactions force chirality flips on internal fermion lines and thus the diagram would vanish if both fermions were massless. Evaluation of the diagrams with W and would-be Goldstones leads to gauge invariant and finite amplitude. Then we find that $\Delta(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$ generates two effective coefficients:

$$
\begin{aligned}
a(d_\alpha, e^C_\beta) = &-\frac{G_F}{4\pi^2 m_W^2} \sum_{j,k} \left[U_C^\dagger (Y^{10*} - Y^{10\dagger}) U_C^*\right]_{1j} \left[D_C^\dagger Y^{\bar{5}\dagger} E_C^*\right]_{k\beta} \\
&\qquad m_{u_j} V_{j\alpha}\, m_{d_k} V^*_{uk}\, J(x_\Delta, x_{u_j}, x_{d_k})\,,
\end{aligned} \tag{53}
$$

$$
\begin{aligned}
a(d_\alpha, d^C_\beta, \nu_i) = &-\frac{G_F}{4\pi^2 m_W^2} \sum_{j} \left[U_C^\dagger (Y^{10*} - Y^{10\dagger}) U_C^*\right]_{1j} \left[D_C^\dagger Y^{\bar{5}\dagger} E_C^*\right]_{\beta i} \\
&\qquad m_{u_j} V_{j\alpha}\, m_{\ell_i}\, J(x_\Delta, x_{u_j}, x_{\ell_i})\,.
\end{aligned} \tag{54}
$$

Here, $V \equiv V_{CKM}$, while the leptonic mixing matrix has been set to unity. (For the neutrino final states one would need to sum over all neutrino flavors.). Mass dependence, apart from helicity flip factors, is encoded in function J (where $x_k \equiv m_k^2/m_W^2$)

$$
\begin{aligned}
J(x, y, z) = &\frac{(y-4) y \log y}{(y-1)(y-x)(y-z)} + \frac{(z-4) z \log z}{(z-1)(z-y)(z-x)} \\
&+ \frac{(x-4) x \log x}{(x-1)(x-y)(x-z)}\,.
\end{aligned} \tag{55}
$$

There are two distinct regimes of dynamics in the box, depending on the presence of t quark in the loop. When $j = 3$ we expand to leading order in $x_{\ell_i}, x_{\ell_i} \ll 1$, and find

$$J(x_\Delta, x_t, x_{\ell_i}) = \frac{1}{x_\Delta - x_t}\left[\frac{x_\Delta - 4}{x_\Delta - 1}\log x_\Delta - \frac{x_t - 4}{x_t - 1}\log x_t\right] . \tag{56}$$

When both fermions are light compared to W the J function takes the following form

$$J(x_\Delta, x_{u_j}, x_{\ell_i}) = \frac{1}{x_\Delta}\left[\frac{x_\Delta - 4}{x_\Delta - 1}\log x_\Delta + \frac{4}{x_{u_j} - x_{\ell_i}}\left(x_{\ell_i}\log x_{\ell_i} - x_{u_j}\log x_{u_j}\right)\right] . \tag{57}$$

Contributions of the up-quark Yukawa couplings are weighted approximately by $m_{u_j} V_{jd}$ for $j = 2, 3$ that run in the box. The large mass of the t quark comes with small element V_{td} that makes this product of the same magnitude as $m_c V_{cd}$. Similar cancellation between mass and CKM hierarchies occurs for the down-quarks and the weights obey $m_d V_{ud} \sim m_s V_{us} \sim m_b V_{ub}$.

The $(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$ state has been identified as a suitable candidate to explain the anomalous value of the muon magnetic moment. One of the leptoquark couplings $(D_C^\dagger Y^{\bar{5}\dagger} E_C^*)_{i2}$ between the muon and one of the down quarks d_i must be of the order ~ 2, while other two must be $\lesssim 10^{-3}$ to suppress contributions to other down-quark and charged lepton observables [31]. In addition, the CDF and DØ measurements of forward-backward asymmetry in $t\bar{t}$ production can be explained by a large diquark coupling $(U_C^\dagger (Y^{10*} - Y^{10\dagger}) U_C^*)_{31} \sim 2$ to ut quark pair [28]. Additional couplings between uc and ct quark pairs are constrained by charm and top physics processes and their upper bounds are of the order 10^{-1} and 10^{-2}, respectively [32]. Both puzzles can be explained for a mass of the leptoquark of around $400\,$GeV. However, for a light mass and with the abovementioned two large couplings the proton would decay much too quickly to a muon final state via dominant contribution of t quark and one of the down quarks in the box (c.f. Eq. (54)). Therefore, one has to find a second amplitude of equal magnitude and opposite phase in order to achieve cancellation between the first and the second amplitude. As explained in the preceding paragraph, all down quarks in the box that couple to external muon have similar weights that come from loop dynamics and CKM factors. As a result, the hierarchy of d_i contributions to the amplitude follows very closely hierarchy of the leptoquark couplings $(D_C^\dagger Y^{\bar{5}\dagger} E_C^*)_{i2}$ and the required cancellation cannot take place between the different down-quarks in the box. Likewise, cancellation between c and t quarks in the box diagram cannot occur for similar reason.

B. Tree-level dimension-nine operator from $(\bar{\mathbf{3}}, \mathbf{1}, 4/3) \in 45$

The W emission from the up-type quark leads to proton decay amplitudes depicted on Fig. 2. Decays of this type have been already mentioned in Ref. [41]. We focus here on the final state with a single charged lepton whose decay width is most severely bounded experimentally. In this case the following $d = 9$ effective operator is obtained

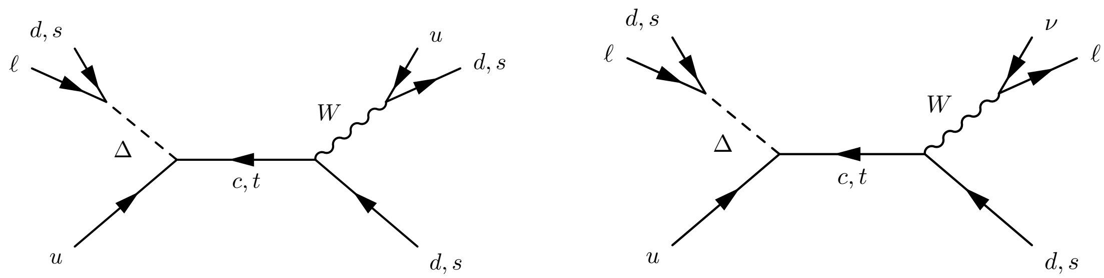

FIG. 2. $d = 6$ proton decay operator induced by tree level $(\bar{\mathbf{3}}, \mathbf{1}, 4/3) \in 45$ and W exchanges.

$$\mathcal{L}_9 = \sum_{U=c,t} \frac{-8G_F V_{U\alpha} V^*_{u\gamma}}{m_U m_\Delta^2} \left[U_C (Y^{10} - Y^{10T})^\dagger U_C^*\right]_{1U} \left[D_C^\dagger Y^{\bar{5}\dagger} E_C^*\right]_{\beta i}$$
$$\epsilon_{abc} (\overline{u_a^C} \gamma^\mu L d_{b\alpha})(\overline{d_{c\beta}^C} R \ell_i)(\overline{d_{k\gamma}} \gamma_\mu L u_k)\,. \tag{58}$$

Here U labels c or t quark, whereas external leptons $i, j = 1, 2$ and down-type quarks $\alpha, \beta, \gamma = 1, 2$, are all light. a, b, c, k are SU(3) color indices. $R(= (1 + \gamma_5)/2)$ is the right projection operator. We focus immediately on best constrained channels, i.e., $p \to \pi^0 \ell_i^+$, and we set $\alpha, \beta, \gamma = 1$. The use of Fierz transformations leads to the amplitude with scalar bilinears

$$\mathcal{M}_9^{p\to\pi^0 e_i^+} = \sum_{U=c,t} \frac{8iG_F}{m_\Delta^2 m_U} \left[U_C (Y^{10} - Y^{10T})^\dagger U_C^*\right]_{1U} \left[D_C^\dagger Y^{\bar{5}\dagger} E_C^*\right]_{1i} V_{Ud} V^*_{ud} \tag{59}$$
$$\epsilon_{abc} \left\langle \pi^0 \ell_i^+ \,\middle|\, (\overline{\ell_i^C} R u_a)\,(\overline{d_k} R d_c)\,(\overline{u_k^C} L d_b) \,\middle|\, p \right\rangle + \text{tensor terms}\,.$$

One can estimate the above matrix element by employing the vacuum saturation approximation. We insert the current $\overline{d_k} R d_c$ between the vacuum and π and end up with product of pion creation and proton annihilation amplitudes. The tensor terms which are invoked by the Fierz relations in Eq. (59) cannot contribute in this case. The vacuum-to-pion amplitude is

$$\left\langle \pi^0 \,\middle|\, \overline{d_k} R d_c \,\middle|\, 0 \right\rangle = \frac{-i m_\pi^2 f_\pi}{4\sqrt{2} m_d} \delta_{ck}\,, \tag{60}$$

whereas the full amplitude is

$$\mathcal{M}_9^{p\to\pi^0 e_i^+} = \sum_{U=c,t} \frac{-\sqrt{2} G_F}{m_\Delta^2 m_U} \left[U_C (Y^{10} - Y^{10T})^\dagger U_C^*\right]_{1U} \left[D_C^\dagger Y^{\bar{5}\dagger} E_C^*\right]_{1i} V_{Ud} V^*_{ud} \frac{m_\pi^2 f_\pi}{m_d} \tag{61}$$
$$\epsilon_{abc} \left\langle \ell_i^+ \,\middle|\, (\overline{u_a^C} L d_b)\,(\overline{\ell_i^C} R u_c) \,\middle|\, p \right\rangle\,.$$

The annihilation matrix element of the proton in Eq. (61) has been most precisely evaluated using lattice QCD [44]. These authors have introduced operators $O_{uds}^{\Gamma\Gamma'} = \epsilon_{abc}(\overline{u_a^C} \Gamma d_b) \Gamma' s_c$ and defined constant α as

$$\alpha R u_p = -\left\langle 0 \,\middle|\, O_{udu}^{LR} \,\middle|\, p \right\rangle\,, \tag{62}$$

where u_p is the Dirac spinor of the proton. The recent value of α obtained from lattice QCD calculation with domain wall fermions [38] is $\alpha = -0.0112(25)\,\mathrm{GeV}^3$. The decay width is then

$$\Gamma(p \to \pi^0 \ell_i^+) = \frac{G_F^2 f_\pi^2 m_\pi^4 \alpha^2}{16\pi m_d^2} \frac{\lambda(m_p^2, m_{\ell_i}^2, m_\pi^2)^{1/2}(m_p^2 + m_{\ell_i}^2 - m_\pi^2)}{m_p^3} \times \left| \sum_{U=c,t} \frac{V_{Ud}\left[U_C(Y^{10} - Y^{10T})^\dagger U_C^*\right]_{1U}\left[D_C^\dagger Y^{\bar{5}\dagger} E_C^*\right]_{1i}}{m_U m_\Delta^2} \right|^2 , \tag{63}$$

where $\lambda(x,y,z) \equiv (x+y+z)^2 - 4(xy+yz+zx)$. From the experimental limits (46) one obtains the following bounds

$$\left| \left[D_C^\dagger Y^{\bar{5}\dagger} E_C^*\right]_{11} \sum_{U=c,t} \frac{V_{U1}\left[U_C(Y^{10} - Y^{10T})^\dagger U_C^*\right]_{1U}}{m_U} \right| < 2.4 \times 10^{-20} \, \frac{m_\Delta^2}{(400\,\mathrm{GeV})^2} \,\mathrm{GeV}^{-1} , \tag{64}$$

$$\left| \left[D_C^\dagger Y^{\bar{5}\dagger} E_C^*\right]_{12} \sum_{U=c,t} \frac{V_{U1}\left[U_C(Y^{10} - Y^{10T})^\dagger U_C^*\right]_{1U}}{m_U} \right| < 2.6 \times 10^{-20} \, \frac{m_\Delta^2}{(400\,\mathrm{GeV})^2} \,\mathrm{GeV}^{-1} . \tag{65}$$

A comment is in order how phenomenologically preferred values of leptoquark and diquark couplings cope with the above constraints. Couplings to the electrons should be small and are in particular not bounded from below, so the constraint from $\tau(p \to \pi^0 e^+)$ can be avoided by putting $\left[D_C^\dagger Y^{\bar{5}\dagger} E_C^*\right]_{11}$ effectively to zero. On the contrary, low-energy leptoquark constraints, especially the $(g-2)_\mu$, indicate that $\left[D_C^\dagger Y^{\bar{5}\dagger} E_C^*\right]_{12}$ could be large in some scenarios [31]. In this case we must require cancellation between the c and t quark amplitudes that occurs when

$$\frac{\left[U_C(Y^{10} - Y^{10T})^\dagger U_C^*\right]_{12}}{\left[U_C(Y^{10} - Y^{10T})^\dagger U_C^*\right]_{13}} \approx -\frac{V_{td}}{V_{cd}} \frac{m_c}{m_t} \approx 2.7 \times 10^{-4} \times e^{-0.37i} . \tag{66}$$

This can be achieved since the $\left[U_C(Y^{10} - Y^{10T})^\dagger U_C^*\right]_{12}$ is only bounded from above while at the same time $\left[U_C(Y^{10} - Y^{10T})^\dagger U_C^*\right]_{13}$ is bounded from below to satisfy observations in $t\bar{t}$ production. Finally, relative phase between the two couplings can be freely adjusted since it is not probed by any experimental observable to date.

Finally, for the state $(\bar{\mathbf{3}}, \mathbf{1}, -2/3)^{+6}$ present in the flipped $SU(5)$ framework we can easily adapt the results obtained above since the two states are indistinguishable at low energies, provided we make the following substitutions

$$\left[U_C^\dagger (Y^{10*} - Y^{10\dagger}) U_C^*\right] \to \frac{1}{2}\left[U_C^\dagger Y^{\bar{5}\dagger} U_C^*\right] , \qquad \left[D_C^\dagger Y^{\bar{5}\dagger} E_C^*\right] \to -\left[D_C^\dagger Y^{1*} E_C^*\right] . \tag{67}$$

To conclude, we note that despite the absence of the tree-level contribution to proton decay of the $(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$ state, weak corrections lead to proton destabilizing $d = 6$ and $d = 9$ operators. The effect of the $d = 9$ operators can be rendered adequately small even in the case of simultaneously large leptoquark *and* diquark couplings, a situation that is favored by observables in $t\bar{t}$ production and value of $(g-2)_\mu$. This is achieved by finely-tuned cancellation of two amplitudes. To the contrary, similar cancellation is impossible in the case of $d = 6$ operator for $p \to \pi^0\mu^+$ decay and we are required to suppress either all leptoquark couplings involving μ or all diquark couplings. We conclude that the proton decay lifetime constraint allows to fully address either $A_{FB}^{t\bar{t}}$ or $(g-2)_\mu$ observable with the $(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$ state, but not both.

VII. CONCLUSIONS

Lepton and baryon number violating interactions are inherently present within grand unified theories and are most severely constrained by the observed proton stability. Proton decay can be mediated by vector or scalar leptoquarks that violate both baryon and lepton number by one unit. Vector leptoquarks that mediate proton decay have gauge couplings to fermions and are not readily allowed to be far below the unification scale. For the scalar leptoquarks, however, the freedom in Yukawa couplings gives one more maneuverability to realize scenarios with light scalar states. On the other hand, the very same Yukawas that are responsible for proton decay very often need to account for the observed fermion mass spectrum. An example of a setting with light leptoquark states was presented in [15, 31, 32] where the low mass of the state $(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$ had an impact on low-energy flavor phenomenology. Most notably, it was found that by tuning independently the two sets of Yukawa couplings, namely the leptoquark and diquark Yukawas, one could reconcile the measured value of forward-backward asymmetry in $t\bar{t}$ production and the value of the magnetic moment of muon.

In this work, we have classified the scalar leptoquarks present in $SU(5)$ and flipped $SU(5)$ grand unification frameworks that mediate proton decay. In both frameworks the considered leptoquark states reside in scalar representations of $SU(5)$ of dimension 5, 10, 45, or 50. We integrate out the above states at tree-level and parameterize their contributions in terms of effective coefficients of a complete set of dimension-six effective operators. The mass constraint on the color triplet state contained in the 5- and 45-dimensional representations is then derived. The precise lower bound depends on the value of the vacuum expectation values of these representation. For the vacuum expectation value of $100\,\text{GeV}$ the lower bound on the triplet mass is approximately 3×10^{12} GeV. The corresponding bound is derived within the flipped $SU(5)$ framework which proves to be less constraining, even in the most conservative case.

The two leptoquark states that do not contribute to proton decay at tree-level are $(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$ and $(\bar{\mathbf{3}}, \mathbf{1}, -2/3)^{+6}$ in the standard and flipped $SU(5)$ frameworks, respectively. We have estimated their contribution to dimension-six operators via box diagram and the tree-level contribution to dimension-nine operators. For the $(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$ state it has been found that if it is to explain both the anomalous magnetic moment of the muon and the $t\bar{t}$ forward-backward asymmetry, then the contribution of the dimension-six operator would destabilize the proton in $p \to \mu^+\pi^0$ channel. Therefore only one of the two puzzles can be addressed with this leptoquark state.

Light scalar leptoquarks can be either produced in pairs or in association with SM fermions at the LHC and are a subject of leptoquark and diquark resonance searches [6, 45–47]. To conclude, we can expect to find signals of these leptoquark states at the LHC, although it seems very unlikely, in light of the constraints from the proton lifetime measurements, that they would be observed in a baryon number violating processes.

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