Collins asymmetry in field ionisation of hydrogen

X. Artru, E. Redouane-Salah

COLLINS ASYMMETRY IN FIELD IONIZATION OF HYDROGEN[1]

X. Artru[1 †] and E. Redouane-Salah[2 †]

(1) *Université de Lyon, CNRS/IN2P3 and Université Lyon 1, IPNL, France*
(2) *Université de M'sila, Faculté des Sciences, Département de Physique and LPMPS, Université de Constantine 1, Algeria*
† *E-mail: x.artru@ipnl.in2p3.fr*

Abstract

An effect similar to the Collins asymmetry is found in the ionization of a hydrogen atom by a static electric field $\mathbf{E}$. When the initial electron possesses an orbital angular momentum $\langle\mathbf{L}\rangle$ transverse to the field, the mean transverse velocity $\langle\mathbf{v}_\mathrm{T}\rangle$ of the final electron points in the direction of $\mathbf{E}\times\langle\mathbf{L}\rangle$. However $\langle\mathbf{L}\rangle$ is oscillating in time due to the linear Stark effect, making $\langle\mathbf{v}_\mathrm{T}\rangle$ oscillate.

Introduction. An atom can be ionized by a sufficiently strong static electric field $\mathbf{E}$ thanks to the tunnel effect. This process has a strong similarity with the production of a quark-antiquark pair ($q\bar{q}$) in a QCD string. If the initial electron has an orbital angular momentum perpendicular to $\mathbf{E}$, the average transverse velocity $\langle\mathbf{v}_\mathrm{T}\rangle$ should be nonzero and in the direction of $\langle\mathbf{L}\rangle\times\mathbf{F}$, where $\mathbf{F}=-e\mathbf{E}$ is the external force [1]. We refer to it as the $\mathbf{v}.(\mathbf{L}\times\mathbf{F})$ asymmetry. The mechanism(Fig.1-left) looks like the *string* + ${}^3P_0$ mechanism (Fig.1-right) of hyperon polarization [2] and Collins effect [1, 3].

At variance with the string + ${}^3P_0$ mechanism, the *Schwinger mechanism* of $q\bar{q}$ pair creation yields no $\mathbf{v}.(\mathbf{L}\times\mathbf{F})$ asymmetry [1]. Thus the question of such an asymmetry in string breaking remains open. It is at least instructive to study it in atomic physics.

1 Behavior of an H atom in an external electric field

We consider an hydrogen atom in an static electric field $\mathbf{E}=-(F/e)\,\hat{\mathbf{z}}$. At small F the *linear* Stark effect just splits the n^{th} energy level in $2n-1$ sublevels separated by[2] $\omega=3nF/2$. *Stark states* are the eigenstates of $H_0-Fz=\mathbf{p}^2/2-1/r-Fz$ in the $F\to 0$ limit. For large enough F ionization by tunneling becomes important and Stark states move into resonances of complex energy $E=E_\mathrm{R}-i\gamma/2$. Using the parabolic coordinates ξ=r-z, η=r+z, φ=arg$(x+iy)$, their wave functions have the separable form [4]

$$\Psi=\xi^{-1/2}\,\Phi(\xi)\;\eta^{-1/2}\,\chi(\eta)\,e^{im\varphi} \tag{1}$$

where $\Phi(\xi)$ verifies

$$\partial^2\Phi/\partial\xi^2+\left[E/2+Z_\xi/\xi-(m^2-1)/(4\xi^2)-F\xi/4\right]\Phi(\xi)=0\,. \tag{2}$$

and $\chi(\eta)$ an analogous equation with $F\to -F$ and $Z_\xi\to Z_\eta=1-Z_\xi$. Stark states are labeled $|n_\xi,n_\eta,m\rangle$, where n_ξ and n_η are the numbers of nodes of $\Phi(\xi)$ and $\chi(\eta)$, linked

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[2]In this paper we use atomic units: $\hbar/(m_e\alpha c)=0.0529$ nm for length, $\hbar/(m_e\alpha^2c^2)=2.42\;10^{-17}$ s for time, $m_e\alpha^2c^2=27,2$ eV for energy and $m_e^2\alpha^3c^2/\hbar=5.14\;10^9$ eV/cm for force.

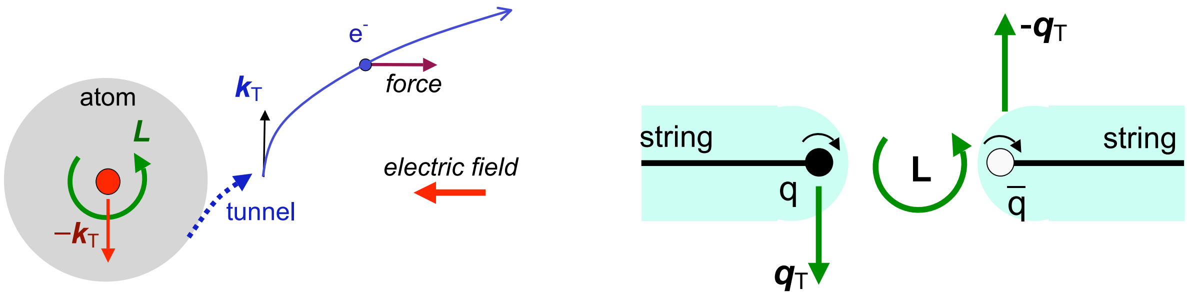

Figure 1: Left: semi-classical motion of the electron extracted form the hydrogen atom by a strong field $\mathbf{E}$, when the electron is initally in a $L_y = +1$ state. Right: String + ${}^3\mathrm{P}_0$ mechanism correlating the tranverse momentum and the transverse polarization of a quark created in string decay [1, 3].

by $n_\eta + n_\xi + |m| + 1 = n$ and fixing $Z_\xi = (n + n_\xi - n_\eta)/(2n)$. With the change of variables $\sqrt{\xi/n}\, e^{i\varphi} = \hat{x} + i\hat{y}$, $\hat{\Phi}(\hat{x}, \hat{y}) \equiv \xi^{-1/2}\Phi(\xi)\, e^{im\varphi}$ is the wave function of a 2-dimensional harmonic oscillator of angular momentum m and energy $\epsilon_\xi = 2nZ_\xi = 2n_\xi + |m| + 1$.

$\mathbf{L}_\perp$ oscillations. Stark states are also eigenstates of A_z, where $\mathbf{A}$ is the Laplace-Runge-Lenz-Pauli vector

$$\mathbf{A} = \mathbf{r}/r + (\mathbf{L} \times \mathbf{p} - \mathbf{p} \times \mathbf{L})/2\,. \tag{3}$$

For $F{=}0$, $\langle A_z \rangle{=}2\langle z \rangle/(3n^2) = (n_\eta - n_\xi)/n$. For $F{\neq}0$ the transverse components (L_x, L_y) and (A_x, A_y) are not conserved. Starting from a L_y eigenstate, $\langle L_y \rangle$ oscillates in quadrature with $\langle A_x \rangle$, as pictured in Fig.2, with the period $2\pi/\omega$. Let us take as an example the initial state $|n{=}2, L_y{=}{+}1\rangle$, whose wave function is

$$\Psi(\mathbf{r}, t{=}0) = 8^{-1}\pi^{-1/2}\,(z + ix)\,e^{-r/2} = 0.5(\,|010\rangle - |100\rangle + i|001\rangle + i|00-1\rangle\,)\,. \tag{4}$$

At $t \neq 0$ it evolves as

$$\Psi(t) \;=\; 0.5\,e^{it/8}\left(e^{+i\omega t}\,|010\rangle - e^{-i\omega t}\,|100\rangle\, + i|001\rangle + i|00-1\rangle\right) \tag{5}$$

$$\;=\; e^{it/8}\left[\cos^2\frac{\omega t}{2}\,|L_y{=}{+}1\rangle - \sin^2\frac{\omega t}{2}\,|L_y{=}{-}1\rangle + \frac{i}{\sqrt{2}}\sin(\omega t)|l{=}0\rangle\right]. \tag{6}$$

Thus the atom oscillates between three L_y eigenstates.

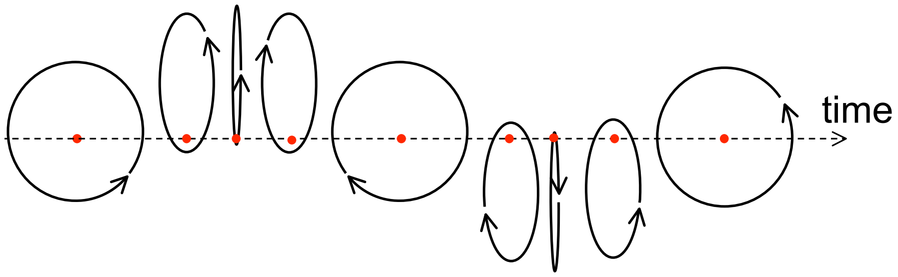

Figure 2: Classical picture of the Stark oscillations of L_y and A_x.

2 Tunneling amplitudes

The external force is confining in ξ and changes $\Phi(\xi)$ only little. Tunneling bears on $\chi(\eta)$. The wave function at large η describes the escaped electron. Using, as in Ref. [4], the JWKB method to lowest order in F, one obtains for the state $|i\rangle \equiv |n_\xi, n_\eta, m\rangle$

$$\Psi_i(\mathbf{r},t)_{\,\eta\to\infty} \simeq a_i\,\hat{\Phi}_i(\hat{x},\hat{y})\exp\{(-i\delta E_i - \gamma_i/2)t'\}\,B(\eta,t)\,. \tag{7}$$

a_i is the tunneling amplitude normalized to $|a_i|^2 = \gamma_i$, δE_i is the Stark shift, $\hat{\Phi}_i(\hat{x},\hat{y})$ is the 2-D oscillator wave function normalized to $\langle\hat{\Phi}_i|\hat{\Phi}_i\rangle = 1$ and

$$B(\eta,t) = (4F\eta^3)^{-1/4}\,\exp\left[(i/3)\sqrt{F}(\eta-\eta_F)^{3/2} + it/8 + 5i\pi/4\right]. \tag{8}$$

$\eta_F \equiv 1/(n^2F)$ is near the tunnel exit and $t' \equiv t - \sqrt{(\eta-\eta_F)/F}$ is the classical electron exit time. For n=2 the amplitudes are

$$\begin{aligned} a_1 &\equiv a_{010} = iq\,a_{00+1}\,,\\ a_2 &\equiv a_{00+1} = a_{00-1} \;=\; 2^{-5/2}F^{-1}\;\exp\left[-1/(24F)\right],\\ a_3 &\equiv a_{100} = \;a_{00+1}/(iq)\,, \end{aligned} \tag{9}$$

with $q = e^{-3/2}/\sqrt{2F}$. The widths $\gamma_i = |a_i|^2$ agree with Slavjanov's result [5].

3 v.(L×F) asymmetry for the initial state $|n{=}2, L_y{=}{+}1\rangle$

With the initial state (4) the escaped electron density is, according to (5,7-8),

$$|\Psi(\mathbf{r},t)|^2_{\eta\to\infty} = (4F\eta^3)^{-1/2}\left|\hat{\Phi}(\hat{x},\hat{y},t')\right|^2 \tag{10}$$

with $\eta \simeq 2z$, $(\hat{x},\hat{y}) \simeq (x,y)/\sqrt{2nz}$. In the n=2 case,

$$\hat{\Phi}(t') = 0.5\left\{a_1\,\hat{\Phi}_{010}\,e^{(i\omega-\gamma_1/2)t'} - a_3\,\hat{\Phi}_{100}\,e^{(-i\omega-\gamma_3/2)t'} + ia_2\,(\hat{\Phi}_{00+1}+\hat{\Phi}_{00-1})\,e^{-\gamma_2 t'/2}\right\}. \tag{11}$$

$|\Psi(\mathbf{r},t)|^2$ looks like the density of a classical electron cloud falling freely in the force field $\mathbf{F}$. An electron leaving the tunnel at time t' with the transverse velocity $\mathbf{v}_\perp$ follows the parabola of fixed $(\hat{x},\hat{y}) \simeq \mathbf{v}_\perp/\sqrt{2F}$. The interference between even- and odd-m terms of $\hat{\Phi}(t')$ yields the **v.(L×F)** asymmetry, which is t'-dependent. A measure of it is

$$A(t') \equiv \langle v_x\rangle/\Delta v_x = \langle\hat{\Phi}(t')|\hat{x}|\hat{\Phi}(t')\rangle/\sqrt{\langle\hat{\Phi}(t')|\hat{x}^2|\hat{\Phi}(t')\rangle}\,; \tag{12}$$

$$A(t'{=}0) = \;8^{1/2}\left(q^2+8+3\,q^{-2}\right)^{-1/2}\,. \tag{13}$$

Like L_y, $A(t')$ changes sign at the Stark frequency, giving the "crawling snake" of Fig.3 [6].

Conclusion. This study shows that the **v.(L×F)** effect does exist in field ionisation, but is oscillating in time. Several constraints make its search challenging:

- Radiative transition may compete with field ionization.
- The initial asymmetry $A(0)$ is small if the $|a_i|$'s differ too much (see Eqs.13 and 9).

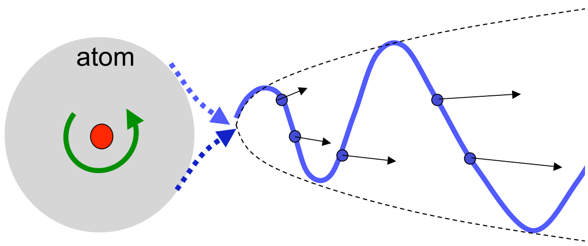

Figure 3: "Crawling snake" motion of $\langle x \rangle$ versus z of the escaping electron. As t grows the undulations move to the right.

- $A(t')$ is fast oscillating, therefore one may only measure its time-averaged $\langle A \rangle$. This one is large only if $\gamma_i \gtrsim \omega$, so that ionization is faster than oscillation.

These constraints are satisfied with a large enough field. In the n=2 case this field is too strong to be produced in laboratory. Hopefully, our results can be generalized to large n (Rydberg states), where the required field scales like n^{-4} [7]. The $\mathbf{v}_\mathrm{T}$ distribution can be measured by the *photoelectron imaging* techniques [8, 9].

Our formulae, obtained at lowest order in F, cannot be applied at the required field. Accurate numerical methods are given in [10, 11]. Nevertheless the above conclusions should remain qualitatively correct.

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