



HAL
open science

Collins asymmetry in field ionisation of hydrogen

X. Artru, E. Redouane-Salah

► **To cite this version:**

X. Artru, E. Redouane-Salah. Collins asymmetry in field ionisation of hydrogen. XV Advanced Research Workshop on High Energy Spin Physics, Oct 2013, Dubna, Russia. pp.41-44. in2p3-00953551

HAL Id: in2p3-00953551

<https://hal.in2p3.fr/in2p3-00953551>

Submitted on 28 Feb 2014

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

COLLINS ASYMMETRY IN FIELD IONIZATION OF HYDROGEN¹

X. Artru^{1†} and E. Redouane-Salah^{2†}

(1) *Université de Lyon, CNRS/IN2P3 and Université Lyon 1, IPNL, France*

(2) *Université de M'sila, Faculté des Sciences, Département de Physique
and LPMPS, Université de Constantine 1, Algeria*

† *E-mail: x.artru@ipnl.in2p3.fr*

Abstract

An effect similar to the Collins asymmetry is found in the ionization of a hydrogen atom by a static electric field \mathbf{E} . When the initial electron possesses an orbital angular momentum $\langle \mathbf{L} \rangle$ transverse to the field, the mean transverse velocity $\langle \mathbf{v}_T \rangle$ of the final electron points in the direction of $\mathbf{E} \times \langle \mathbf{L} \rangle$. However $\langle \mathbf{L} \rangle$ is oscillating in time due to the linear Stark effect, making $\langle \mathbf{v}_T \rangle$ oscillate.

Introduction. An atom can be ionized by a sufficiently strong static electric field \mathbf{E} thanks to the tunnel effect. This process has a strong similarity with the production of a quark-antiquark pair ($q\bar{q}$) in a QCD string. If the initial electron has an orbital angular momentum perpendicular to \mathbf{E} , the average transverse velocity $\langle \mathbf{v}_T \rangle$ should be nonzero and in the direction of $\langle \mathbf{L} \rangle \times \mathbf{F}$, where $\mathbf{F} = -e\mathbf{E}$ is the external force [1]. We refer to it as the $\mathbf{v} \cdot (\mathbf{L} \times \mathbf{F})$ asymmetry. The mechanism (Fig.1-left) looks like the *string* + 3P_0 mechanism (Fig.1-right) of hyperon polarization [2] and Collins effect [1, 3].

At variance with the string + 3P_0 mechanism, the *Schwinger mechanism* of $q\bar{q}$ pair creation yields no $\mathbf{v} \cdot (\mathbf{L} \times \mathbf{F})$ asymmetry [1]. Thus the question of such an asymmetry in string breaking remains open. It is at least instructive to study it in atomic physics.

1 Behavior of an H atom in an external electric field

We consider an hydrogen atom in an static electric field $\mathbf{E} = -(F/e)\hat{\mathbf{z}}$. At small F the *linear* Stark effect just splits the n^{th} energy level in $2n - 1$ sublevels separated by² $\omega = 3nF/2$. *Stark states* are the eigenstates of $H_0 - Fz = \mathbf{p}^2/2 - 1/r - Fz$ in the $F \rightarrow 0$ limit. For large enough F ionization by tunneling becomes important and Stark states move into resonances of complex energy $E = E_R - i\gamma/2$. Using the parabolic coordinates $\xi=r-z$, $\eta=r+z$, $\varphi=\arg(x+iy)$, their wave functions have the separable form [4]

$$\Psi = \xi^{-1/2} \Phi(\xi) \eta^{-1/2} \chi(\eta) e^{im\varphi} \quad (1)$$

where $\Phi(\xi)$ verifies

$$\partial^2 \Phi / \partial \xi^2 + [E/2 + Z_\xi/\xi - (m^2 - 1)/(4\xi^2) - F\xi/4] \Phi(\xi) = 0. \quad (2)$$

and $\chi(\eta)$ an analogous equation with $F \rightarrow -F$ and $Z_\xi \rightarrow Z_\eta = 1 - Z_\xi$. Stark states are labeled $|n_\xi, n_\eta, m\rangle$, where n_ξ and n_η are the numbers of nodes of $\Phi(\xi)$ and $\chi(\eta)$, linked

¹Presented at XVI Advanced Research Workshop on High Energy Spin Physics (DSPIN-13)(Dubna, October 8-12, 2013)

²In this paper we use atomic units: $\hbar/(m_e\alpha c) = 0.0529$ nm for length, $\hbar/(m_e\alpha^2 c^2) = 2.42 \cdot 10^{-17}$ s for time, $m_e\alpha^2 c^2 = 27,2$ eV for energy and $m_e^2\alpha^3 c^2/\hbar = 5.14 \cdot 10^9$ eV/cm for force.

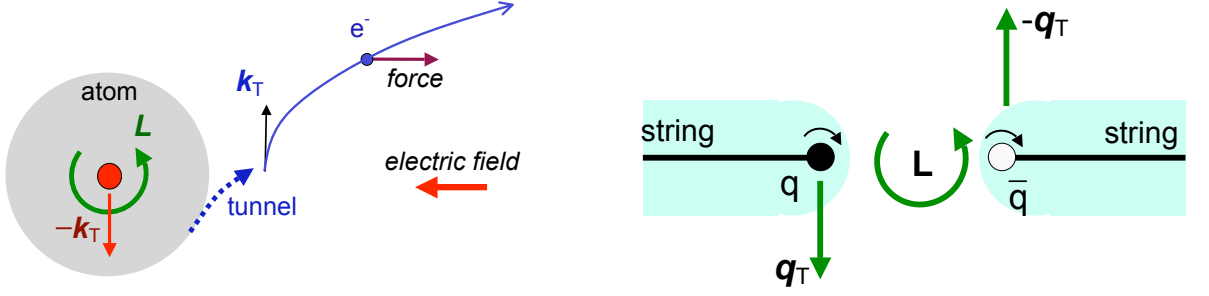


Figure 1: Left: semi-classical motion of the electron extracted from the hydrogen atom by a strong field \mathbf{E} , when the electron is initially in a $L_y = +1$ state. Right: String + 3P_0 mechanism correlating the transverse momentum and the transverse polarization of a quark created in string decay [1, 3].

by $n_\eta + n_\xi + |m| + 1 = n$ and fixing $Z_\xi = (n + n_\xi - n_\eta)/(2n)$. With the change of variables $\sqrt{\xi/n} e^{i\varphi} = \hat{x} + i\hat{y}$, $\hat{\Phi}(\hat{x}, \hat{y}) \equiv \xi^{-1/2} \Phi(\xi) e^{im\varphi}$ is the wave function of a 2-dimensional harmonic oscillator of angular momentum m and energy $\epsilon_\xi = 2nZ_\xi = 2n_\xi + |m| + 1$.

L_\perp oscillations. Stark states are also eigenstates of A_z , where \mathbf{A} is the Laplace-Runge-Lenz-Pauli vector

$$\mathbf{A} = \mathbf{r}/r + (\mathbf{L} \times \mathbf{p} - \mathbf{p} \times \mathbf{L})/2. \quad (3)$$

For $F=0$, $\langle A_z \rangle = 2\langle z \rangle / (3n^2) = (n_\eta - n_\xi)/n$. For $F \neq 0$ the transverse components (L_x, L_y) and (A_x, A_y) are not conserved. Starting from a L_y eigenstate, $\langle L_y \rangle$ oscillates in quadrature with $\langle A_x \rangle$, as pictured in Fig.2, with the period $2\pi/\omega$. Let us take as an example the initial state $|n=2, L_y=+1\rangle$, whose wave function is

$$\Psi(\mathbf{r}, t=0) = 8^{-1} \pi^{-1/2} (z + ix) e^{-r/2} = 0.5(|010\rangle - |100\rangle + i|001\rangle + i|00-1\rangle). \quad (4)$$

At $t \neq 0$ it evolves as

$$\Psi(t) = 0.5 e^{it/8} (e^{+i\omega t} |010\rangle - e^{-i\omega t} |100\rangle + i|001\rangle + i|00-1\rangle) \quad (5)$$

$$= e^{it/8} \left[\cos^2 \frac{\omega t}{2} |L_y=+1\rangle - \sin^2 \frac{\omega t}{2} |L_y=-1\rangle + \frac{i}{\sqrt{2}} \sin(\omega t) |l=0\rangle \right]. \quad (6)$$

Thus the atom oscillates between three L_y eigenstates.

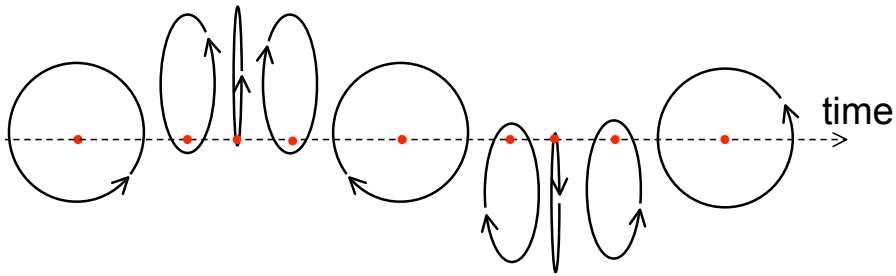


Figure 2: Classical picture of the Stark oscillations of L_y and A_x .

2 Tunneling amplitudes

The external force is confining in ξ and changes $\Phi(\xi)$ only little. Tunneling bears on $\chi(\eta)$. The wave function at large η describes the escaped electron. Using, as in Ref. [4], the JWKB method to lowest order in F , one obtains for the state $|i\rangle \equiv |n_\xi, n_\eta, m\rangle$

$$\Psi_i(\mathbf{r}, t)_{\eta \rightarrow \infty} \simeq a_i \hat{\Phi}_i(\hat{x}, \hat{y}) \exp\{(-i\delta E_i - \gamma_i/2)t'\} B(\eta, t). \quad (7)$$

a_i is the tunneling amplitude normalized to $|a_i|^2 = \gamma_i$, δE_i is the Stark shift, $\hat{\Phi}_i(\hat{x}, \hat{y})$ is the 2-D oscillator wave function normalized to $\langle \hat{\Phi}_i | \hat{\Phi}_i \rangle = 1$ and

$$B(\eta, t) = (4F\eta^3)^{-1/4} \exp\left[(i/3)\sqrt{F}(\eta - \eta_F)^{3/2} + it/8 + 5i\pi/4\right]. \quad (8)$$

$\eta_F \equiv 1/(n^2F)$ is near the tunnel exit and $t' \equiv t - \sqrt{(\eta - \eta_F)/F}$ is the classical electron exit time. For $n=2$ the amplitudes are

$$\begin{aligned} a_1 &\equiv a_{010} = iq a_{00+1}, \\ a_2 &\equiv a_{00+1} = a_{00-1} = 2^{-5/2} F^{-1} \exp[-1/(24F)], \\ a_3 &\equiv a_{100} = a_{00+1}/(iq), \end{aligned} \quad (9)$$

with $q = e^{-3/2}/\sqrt{2F}$. The widths $\gamma_i = |a_i|^2$ agree with Slavjanov's result [5].

3 $\mathbf{v} \cdot (\mathbf{L} \times \mathbf{F})$ asymmetry for the initial state $|n=2, L_y = +1\rangle$

With the initial state (4) the escaped electron density is, according to (5,7-8),

$$|\Psi(\mathbf{r}, t)|_{\eta \rightarrow \infty}^2 = (4F\eta^3)^{-1/2} \left| \hat{\Phi}(\hat{x}, \hat{y}, t') \right|^2 \quad (10)$$

with $\eta \simeq 2z$, $(\hat{x}, \hat{y}) \simeq (x, y)/\sqrt{2nz}$. In the $n=2$ case,

$$\hat{\Phi}(t') = 0.5 \left\{ a_1 \hat{\Phi}_{010} e^{(i\omega - \gamma_1/2)t'} - a_3 \hat{\Phi}_{100} e^{(-i\omega - \gamma_3/2)t'} + ia_2 (\hat{\Phi}_{00+1} + \hat{\Phi}_{00-1}) e^{-\gamma_2 t'/2} \right\}. \quad (11)$$

$|\Psi(\mathbf{r}, t)|^2$ looks like the density of a classical electron cloud falling freely in the force field \mathbf{F} . An electron leaving the tunnel at time t' with the transverse velocity \mathbf{v}_\perp follows the parabola of fixed $(\hat{x}, \hat{y}) \simeq \mathbf{v}_\perp/\sqrt{2F}$. The interference between even- and odd- m terms of $\hat{\Phi}(t')$ yields the $\mathbf{v} \cdot (\mathbf{L} \times \mathbf{F})$ asymmetry, which is t' -dependent. A measure of it is

$$A(t') \equiv \langle v_x \rangle / \Delta v_x = \langle \hat{\Phi}(t') | \hat{x} | \hat{\Phi}(t') \rangle / \sqrt{\langle \hat{\Phi}(t') | \hat{x}^2 | \hat{\Phi}(t') \rangle}; \quad (12)$$

$$A(t'=0) = 8^{1/2} (q^2 + 8 + 3q^{-2})^{-1/2}. \quad (13)$$

Like L_y , $A(t')$ changes sign at the Stark frequency, giving the "crawling snake" of Fig.3 [6].

Conclusion. This study shows that the $\mathbf{v} \cdot (\mathbf{L} \times \mathbf{F})$ effect does exist in field ionisation, but is oscillating in time. Several constraints make its search challenging:

- Radiative transition may compete with field ionization.
- The initial asymmetry $A(0)$ is small if the $|a_i|$'s differ too much (see Eqs.13 and 9).

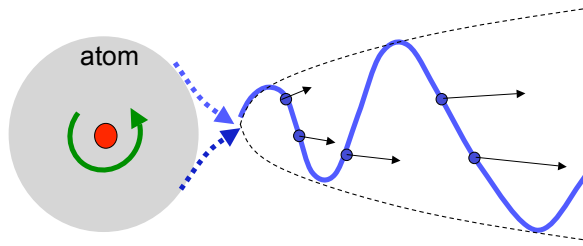


Figure 3: "Crawling snake" motion of $\langle x \rangle$ versus z of the escaping electron. As t grows the undulations move to the right.

- $A(t')$ is fast oscillating, therefore one may only measure its time-averaged $\langle A \rangle$. This one is large only if $\gamma_i \gtrsim \omega$, so that ionization is faster than oscillation.

These constraints are satisfied with a large enough field. In the $n=2$ case this field is too strong to be produced in laboratory. Hopefully, our results can be generalized to large n (Rydberg states), where the required field scales like n^{-4} [7]. The \mathbf{v}_T distribution can be measured by the *photoelectron imaging* techniques [8, 9].

Our formulae, obtained at lowest order in F , cannot be applied at the required field. Accurate numerical methods are given in [10, 11]. Nevertheless the above conclusions should remain qualitatively correct.

References

- [1] X. Artru and J. Czyzewski, *Acta Phys. Polonica C* **B29** (1998) 2015.
- [2] B. Andersson, G. Gustafson, G. Ingelman and T. Sjöstrand, *Phys. Rep.* **97** (1983) 31.
- [3] X. Artru, J. Czyzewski and H. Yabuki, *Zeit. Phys. C* **73** (1997) 527.
- [4] L.D. Landau, E.M. Lifshitz, *Course of theoretical physics, Vol. 3, Quantum Mechanics*, Pergamon press, London.
- [5] Yu. Slavjanov, *Problemi Matematicheskoi Fiziki* (Leningrad: Lenigrad State University, 1970), pp 125-34.
- [6] E. Redouane-Salah and X. Artru, *AIP Conf. Proc.* **1444**, 157 (2012).
- [7] X. Artru and E. Redouane-Salah, in preparation.
- [8] Yu.N. Demkov, V.D. Kondratovich and V.N. Ostrovskii, *Pis'ma Zh. Eksp. Teor. Fiz.* **34**, 425 (1981) [*JETP Lett.* **34**, 403 (1981)].
- [9] A.S. Stodolna, A. Rouze, F. Lépine, S. Cohen, F. Robicheaux, A. Gijsbertsen, J.H. Jungmann, C. Bordas and M.J.J. Vrakking, *Phys. Rev. Lett.* **110**, 213001 (2013).
- [10] R.J. Damburg and V.V. Kolosov, *J. Phys. B* **9**, 3149 (1976), **B 11**, 1921 (1978), **B 12**, 2637 (1979).
- [11] T. Yamabe, A. Tachibana and H.J. Silverstone, *Phys. Rev. A* **16** (1977) 877.