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EPJ Nuclear Sciences & Technologies

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Monte Carlo MSM correction factors for control rod worth estimates in subcritical and near-critical fast neutron reactors

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Abstract. The GUINEVERE project was launched in 2006, within the 6th Euratom Framework Program IP-EUROTRANS, in order to study the feasibility of transmutation in Accelerator Driven subcritical Systems (ADS). This zero-power facility hosted at the SCK·CEN site in Mol (Belgium) couples the fast subcritical lead reactor VENUS-F with an external neutron source provided by interaction of deuterons delivered by the GENEPI-3C accelerator and a tritiated target located at the reactor core center. In order to test on-line subcriticality monitoring techniques, the reactivity of all the VENUS-F configurations used must be known beforehand to serve as benchmark values. That is why the Modified Source Multiplication Method (MSM) is under consideration to estimate the reactivity worth of the control rods when the reactor is largely subcritical as well as near-critical. The MSM method appears to be a technique well adapted to measure control rod worth over a large range of subcriticality levels. The MSM factors which are required to account for spatial effects in the reactor can be successfully calculated using a Monte Carlo neutron transport code.

1 Introduction

The GUINEVERE (Generator of Uninterrupted Intense NEutrons at the lead VEnus REactor) project [1] was launched in 2006, within the 6th Euratom Framework Program IP-EUROTRANS [2], in order to study the feasibility of transmutation in Accelerator Driven subcritical Systems (ADS). This facility hosted at the SCK·CEN site in Mol (Belgium) is presently used in the follow-up FREYA project (7th European FP) [3]. It couples the fast subcritical lead-moderated reactor VENUS-F with an external neutron source provided by the deuteron accelerator GENEPI-3C via $T(d,n)^4He$ fusion reactions occurring at the reactor core center (Fig. 1). It is partially dedicated to the investigation of techniques of on-line subcriticality monitoring.

The VENUS-F reactor core is very modular and its reactivity can range from deep subcritical to critical by varying the number of fuel assemblies loaded in the core. It is

also equipped with two boron carbide control rods which allow for a finer tuning of the reactivity. Fission chambers, spread throughout the reactor, allow recording count rates during either steady-state or time-dependent measurements.

In order to test on-line subcriticality monitoring techniques, the reactivity of all the VENUS-F configurations used must be known beforehand to serve as benchmark values. Thus, the reactivity worth of the control rods must be known as accurately as possible so that the reactivity of every new reactor configuration created by moving the control rods be estimated correctly.

Although the reactor asymptotic period measurement is a usual technique to determine the reactivity worth of control rods, it is limited to a small reactivity range (from ≈ –0.3 $ to +0.3 $) [4]. Consequently, it does not always allow measuring the total reactivity worth of the control rods. Furthermore, it is obviously inapplicable to control rod worth measurement in deep subcritical reactors.

This is the reason why the Modified Source Multiplication Method (MSM) [5] is under consideration to be used as an

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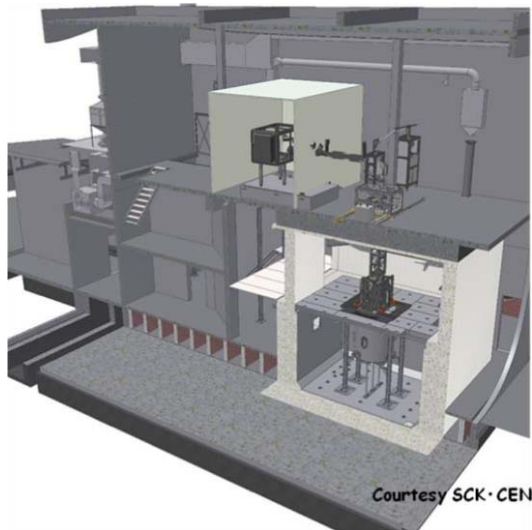
Fig. 1. Overview of the GUINEVERE facility at SCK·CEN.

alternative method for estimating the reactivity worth of the VENUS-F control rods when the reactor is largely subcritical as well as near-critical. In this technique, the unknown reactivity is determined by comparing detector count rates driven by an external neutron source in the configuration of interest (in this paper it will be a new configuration obtained by moving the control rods) with those obtained with the same neutron source in another subcritical configuration whose reactivity is already known (reference configuration). However, to account for the flux shape differences between the two reactor configurations, some position-dependent correction factors (the so-called MSM factors) must be calculated using a neutron transport code.

In this paper, we first present the GUINEVERE facility and the various configurations of the VENUS-F reactor studied. Then the principle of the MSM method is briefly exposed. The results of MSM factor calculations performed with the Monte Carlo neutron transport code MCNP are also shown. They were carried out in support to MSM experiments dedicated to the measurement of the VENUS-F control rod worth when the reactor was either subcritical or near critical. In the former case, the GENEPI-3C was used to generate the neutron external source. In the latter case, an Am-Be neutron source was inserted in the reactor.

General trends in the MSM factor behavior which depend on the neutron source and detector locations, as well as on the reactor subcriticality level are outlined. Finally the calculated MSM correction factors are applied to the detector count rates measured during the MSM experiments. The consistency between the reactivity values given by the detectors is discussed.

2 The GUINEVERE facility

2.1 The VENUS-F reactor

The VENUS-F fast reactor is contained in a cylindrical vessel of approximately 80 cm in radius and 140 cm in

height. A 12×12 grid surrounded by a square stainless steel casing can receive up to 144 elements of $\approx 8 \times 8$ cm^2 in section which can be fuel assemblies, lead assemblies or specific elements for accommodating detectors or absorbent rods. The remaining room in the vessel is filled with semi-circular lead plates, which act as a radial neutron reflector. In addition, the core is equipped with top and bottom 40 cm-thick lead reflectors. Each fuel assembly (FA) contains a 5×5 pattern, filled with 9 fuel rodlets and 16 lead bars, surrounded by lead plates. The fuel is 30 wt.% enriched metallic uranium provided by CEA. Among the set of FAs, six are actually safety rods (SR) made of boron carbide and fuel followers with the absorbent part retracted from the core in normal operation. Two control rods (CR) made of natural boron carbide square cuboids can be positioned at various locations in the 12×12 grid. They can be moved vertically from 0 mm (fully inserted in the core) to 600 mm (fully retracted). Another absorbent rod, whose reactivity worth is very small, called PEAR (Pellet Absorber Rod) rod, is available for performing rod drop experiments.

Various configurations of the reactor in terms of reactivity can be studied thanks to the modular shape of the core. In this paper, since we are interested in measuring the reactivity worth of the set of two CRs, all the reactor configurations studied were obtained from either a near-critical reactor configuration called CR0↓ or a subcritical configuration named SC1↓, by moving the two CRs together at various heights. Since the reactivities of the CR0↓ and SC1↓ configurations had been measured during previous experiments [6], they could serve as reference values for applying the MSM method.

The so-called CR0↓ configuration is represented in Figure 2. Ninety-seven FAs (in blue for the regular ones, in light blue for the SRs with fuel followers) are arranged in a way to create a pseudo-cylindrical core. The two boron-carbide CRs (in red) are located at the core periphery and retracted at approximately 515 mm in height. The CR0↓ configuration was created from a critical one by dropping the PEAR rod (in green). After analyzing the rod drop experiments using Inverse Point Kinetics, the reactivity of CR0↓ was found to be –136(2) pcm [6]. As shown in Figure 2, the reactor was equipped with 9 fission chambers (FCs) working in pulse mode. Three different types of FCs were used, either Photonis CFUL01 and CFUM21[1], or GE Reuter-Stokes (RS), whose specifications are listed in Table 1. In order to help localizing the various assemblies and detectors, an arbitrary coordinate system is used in the 12×12 grid: the upper left corner is labeled (–6,6) and the lower right one (6,–6), there is no (0,0) element. Outside the 12×12 grid, six cylindrical cavities bored in the outer reflector can receive experimental devices. They are labeled, from left to right: A1, B1, C1, A2, B2, and C2.

The so-called SC1↓ configuration is shown in Figure 3. It is derived from the CR0↓ configuration by removing the four central FAs. This removal also permits the insertion of the accelerator thimble inside the VENUS-F core.
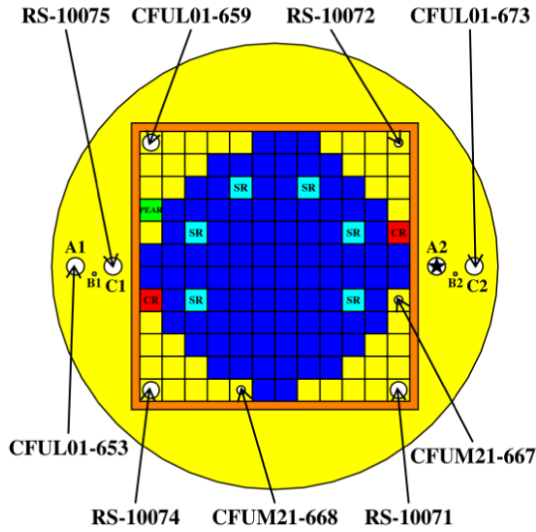
Fig. 2. Schematic view of the CR0↓ configuration. The black star shows the position of the external neutron source (Am-Be). Control rods (CR) are in red.

Fig. 3. Schematic view of the SC1↓ configuration. The black star shows the position of the external neutron source (GENEPI-3C). Control rods (CR) are in red.

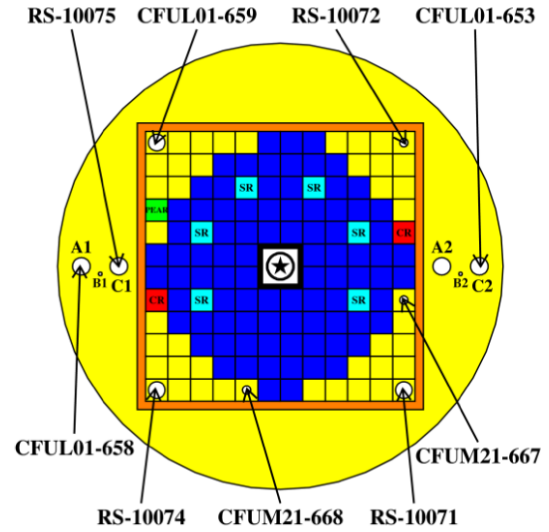Compared to CR0↓, some additional minor differences are itemized below:

– the CRs are slightly more inserted inside the core (CR height is 479 mm instead of 515 mm);
– the detector set is slightly different: the CFUL01-673 detector is replaced by the CFUL01-653 FC. The latter is replaced in the A1 location by the CFUL01-658 FC which is identical to CFUL01-659 and CFUL01-653 FCs. The reactivity of SC1↓ was measured using the MSM method and was found to be –3824(96) pcm [6].

2.2 External neutron sources

The external neutron source used for performing the MSM experiments was different depending on whether the reference configuration was CR0↓ or SC1↓.

Table 1. Fission chambers used in configurations CR0↓ and SC1↓.

Name	Main deposit	Approximate mass (mg)	Location in CR0↓	Location in SC1↓
CFUL01-653	^{235}U	1000	A1	C2
CFUL01-658	^{235}U	1000	None	A1
CFUL01-659	^{235}U	1000	(–6,6)	(–6,6)
CFUL01-673	^{238}U	1000	C2	None
RS-10071	^{235}U	100	(6,–6)	(6,–6)
RS-10072	^{235}U	100	(6,6)	(6,6)
RS-10074	^{235}U	100	(–6,–6)	(–6,–6)
RS-10075	^{235}U	100	C1	C1
CFUM21-667	^{235}U	10	(6,–2)	(6,–2)
CFUM21-668	^{235}U	10	(–2,–6)	(–2,–6)

In the latter configuration, the external source was created at the center of the VENUS-F core by deuterons interacting with a tritiated titanium target. The deuteron ions were accelerated up to an energy of 220 keV by the GENEPI-3C particle accelerator [7] built by a collaboration of CNRS-IN2P3 laboratories. The fusion reactions at core mid-plane generate a quasi-isotropic field of ∼14-MeV neutrons. The GENEPI-3C can operate in pulsed mode, in continuous mode, and also in continuous mode with short beam interruptions. During the MSM experiments reported here, GENEPI-3C delivered a continuous deuteron beam whose intensity ranged from ∼400 μA to ∼500 μA.

However, the intensity of the external neutron source created by the accelerator had to be monitored directly. Indeed, the tritium release and the beam tuning variations over time prevent the direct calculation of the neutron source intensity from that of the beam on target. This is the reason why the accelerator is equipped with two Si detectors which can detect either alpha particles from T $(d,n)^4$He reactions or protons from D(d,p)T reactions. The detection of α particles allows one to quantify the amount of 14-MeV neutrons produced whereas the detection of protons allows estimating the parasitic production of 2.5-MeV neutrons by D$(d,n)^3$He due to the implantation of deuterons in the target. During the MSM experiments reported here, the neutron source intensity varied from ∼1×10^9 to ∼3×10^9 14-MeV neutrons/s.

In the CR0↓ configuration, the external neutron source was an Am-Be source inserted in the outer reflector slot A2 (denoted by a star in Fig. 2) which emitted only 2.2×10^6 neutrons/s. Thus the Am-Be neutron source intensity is lower than that induced by the GENEPI-3C by three orders of magnitude. Furthermore, mainly because the Am-Be source is off-centered, its importance is approximately eight times lower than that of GENEPI-3C. In terms of detector count rates, these source dissimilarities are only (very) partially compensated by the difference in reactivity between the two CR0↓ and SC1↓

configurations. Therefore, since the two reference configurations are very dissimilar both in terms of reactivity and of source location, interesting differences in the results of the MSM experiments can be anticipated.

3 The MSM method

3.1 Principle

The MSM (Modified Source Multiplication) method is a technique for estimating the unknown reactivity of a subcritical configuration by comparing detector count rates driven by an external neutron source in this configuration with those obtained in another subcritical configuration whose reactivity is known.

The inhomogeneous transport equation associated with a subcritical configuration of a reactor driven by an external neutron source reads:

$$A\Phi = P\Phi + S \tag{1}$$

where P is the neutron production operator (by fission or (n, xn) reactions), A is the migration and loss operator and S is the external neutron source intensity. Φ is the neutron flux which is present inside the reactor when the external neutron source is inserted.

This transport equation can be made homogeneous by introducing the neutron multiplication coefficient k_{eff}:

$$A\varphi = \frac{1}{k_{eff}} P\varphi. \tag{2}$$

In that case, φ is the fundamental mode corresponding to the associated critical reactor. $\lambda = 1/k_{eff}$ is also an eigenvalue of the adjoint homogeneous equation:

$$A^{\dagger}\varphi^{\dagger} = \frac{1}{k_{eff}} P^{\dagger}\varphi^{\dagger}. \tag{3}$$

where $A^{\dagger}$ and $P^{\dagger}$ are the adjoint operators of A and P, respectively. $\varphi^{\dagger}$ is the adjoint flux, also called neutron importance function.

Multiplying the adjoint homogeneous equation (3) by Φ and integrating over space, angle and energy, one gets:

$$\rho = \frac{\left\langle \Phi, \left(P^{\dagger} - A^{\dagger}\right)\varphi^{\dagger}\right\rangle}{\left\langle \Phi, P^{\dagger}\varphi^{\dagger}\right\rangle} = \frac{\left\langle \varphi^{\dagger}, (P - A)\Phi\right\rangle}{\left\langle \varphi^{\dagger}, P\Phi\right\rangle} \tag{4}$$

where $\langle\rangle$ denotes such an integration.

Then, multiplying the inhomogeneous equation (1) by $\varphi^{\dagger}$ and integrating over space, angle and energy leads to:

$$\left\langle \varphi^{\dagger}, (P - A)\Phi\right\rangle = -\left\langle \varphi^{\dagger}, S\right\rangle \tag{5}$$

and combining equation (4) and equation (5), one gets:

$$\rho = -\frac{\left\langle \varphi^{\dagger}, S\right\rangle}{\left\langle \varphi^{\dagger}, P\Phi\right\rangle}. \tag{6}$$

As in reference [5], we introduce the reaction rate in the detector $C = \langle \Sigma_d, \Phi\rangle$, where Σ_d is the macroscopic reaction cross-section of the detector, and rewrite equation (6):

$$\rho = -\left\langle \varphi^{\dagger}, S\right\rangle \times \frac{\langle \Sigma_d, \Phi\rangle}{\left\langle \varphi^{\dagger}, P\Phi\right\rangle} \times \frac{1}{\langle \Sigma_d, \Phi\rangle}$$

$$= -S_{eff} \times \varepsilon \times \frac{1}{C} \tag{7}$$

where $S_{eff} = \left\langle \varphi^{\dagger}, S\right\rangle$ is called the effective neutron source and $\varepsilon = \langle \Sigma_d, \Phi\rangle / \left\langle \varphi^{\dagger}, P\Phi\right\rangle$ the detector efficiency.

Now let us consider two subcritical configurations. Let configuration 0 be the subcritical configuration of known reactivity ρ_0 and configuration 1 be that of unknown reactivity ρ_1. Assuming that the neutron external source and the detectors utilised are the same in both configurations, equation (7) can be used to find a relationship between ρ_0, ρ_1, and the detector count rates C_0 and C_1 in configurations 0 and 1:

$$\frac{\rho_1}{\rho_0} = \frac{S_{eff,1}\varepsilon_1}{S_{eff,0}\varepsilon_0} \times \frac{C_0}{C_1} = f_{MSM} \times \frac{C_0}{C_1} \tag{8}$$

where f_{MSM} is the MSM correction factor. One can also introduce the source importance φ which is defined as the ratio of the average importance of external source neutrons to the average importance of fissions in the reactor [8,9]:

$$\varphi = \left(\frac{\left\langle \varphi^{\dagger}, S\right\rangle}{\langle S\rangle}\right)\left(\frac{\left\langle \varphi^{\dagger}, P\Phi\right\rangle}{\langle P\Phi\rangle}\right)^{-1}. \tag{9}$$

If one introduces the source multiplication coefficient k_s as [9]:

$$k_s = \frac{\langle P\Phi\rangle + \langle S\rangle}{\langle S\rangle} \tag{10}$$

the source importance appears as the ratio of the neutron gain with the external neutron source to a hypothetical gain which would be obtained with a stabilized fission source in the same reactor:

$$\varphi = \left(\frac{k_s}{1-k_s}\right)\left(\frac{1-k_{eff}}{k_{eff}}\right). \tag{11}$$

Then the MSM factor can be rewritten with the source importance of the two configurations:

$$f_{MSM} = \frac{\varphi_1}{\varphi_0} \times \frac{e_1}{e_0} \tag{12}$$

where φ_i and $k_{s,i}$ are respectively the source importance and the source multiplication coefficient in configuration i. The parameter e_i is defined as:

$$e_i = \frac{\langle \Sigma_d, \Phi_i\rangle}{\langle P_i\Phi_i\rangle}. \tag{13}$$

It represents the ratio of the reaction rate in the detector to the total rate of neutron produced in the reactor. Thus,

formula (12) shows that the MSM factor accounts for the differences in neutron and source importance as well as in flux shapes between the two configurations considered.

However, if configurations 0 and 1 are very similar, such differences may vanish and formula (8) reduces to the Approximate Source Method (ASM) formula:

$$\frac{\rho_1}{\rho_0} = \frac{k_{eff,0}}{k_{eff,1}}\frac{C_0}{C_1} \approx \frac{C_0}{C_1} \qquad (14)$$

where the approximation $k_{eff,0}/k_{eff,1} \approx 1$ is often made.

The MSM correction factors must be calculated using a transport code, either deterministic or stochastic. It is worth mentioning that the value of the MSM correction factor is expected to depend on the detector location. Indeed, any difference in the flux shape between the two configurations will result in position-dependent ratios in the f_{MSM} formula.

3.2 Calculation of MSM factors

Starting from equation (8), the MSM factor reads:

$$f_{MSM} = \frac{\rho_1}{\rho_0} \times \frac{C_1}{C_0} \qquad (15)$$

where the reactivity of configurations 0 and 1, ρ_0 and ρ_1, as well as the detector count rates in configurations 0 and 1, C_0 and C_1, can be calculated using a neutron transport code.

Although the use of deterministic codes is largely reported in literature, MSM factors can also be calculated using stochastic neutron transport codes. On one hand, the use of a Monte Carlo code advantageously allows one to transport neutrons in the reactor theoretically without any geometry simplification (to the extent that the reactor geometry be accurately known) and with pointwise energy dependent cross-sections. On the other hand, Monte Carlo calculations are much more computer-time-consuming than deterministic ones and provide as results only statistical estimates of quantities of interest. In this paper, the Monte Carlo simulation code MCNP 5 [10] was employed, together with ZZ ALEPH-LIB-JEFF3.1.1, a continuous energy multi-temperature library created at SCK·CEN and based on JEFF3.1.1 [11]. Once the geometry as well as the material composition of the various elements constituting the reactor have been described in an MCNP input file, the corresponding multiplication factor (hence the reactivity) can be estimated using a generation-based, iterative fission neutron source whose spatial distribution converges towards the fundamental mode of the reactor (the so-called "kcode" source). On the other hand, standard fixed-source calculations can provide estimates of reaction rates anywhere in the reactor. So, for the calculation of MSM factors, four Monte Carlo simulations must be run: two fixed-source simulations for calculating the source driven reaction rates C_0 and C_1 in the fission chambers for configurations 0 and 1, and two "kcode" simulations for estimating the reactivity of the same two configurations, ρ_0 and ρ_1.

As a first step towards the calculation of MSM factors, MCNP input files had to be built for the configurations CR0↓ and SC1↓ as well as their variants created by moving the CRs. In order to save computing times (a factor of ∼4.5 was gained), it was decided to use a simplified reactor geometry. Indeed, the MSM method bears interest only if the calculation of MSM factors turns out to be rather insensitive to the details and errors on the reactor geometry, as well as to uncertainties on material compositions and on nuclear data: since MSM experiments are carried out to estimate the unknown reactivity of a reactor configuration, one can imagine that the reactor itself could be not very well known either. Fortunately, this robustness of MSM factor calculations has already been observed for previous MSM experiments at the VENUS-F reactor and can be understood by recalling that MSM factors are double ratios of quantities: one can expect that any reasonable difference between the calculated reactivity values and the real ones will be at least partially compensated by corresponding differences between the calculated reaction rates and the measured ones [6].

Since the control and safety rods are nearly homogeneous, the principal source of geometrical simplification was the homogenization of the fuel assemblies. Additionally, some details of the bottom reactor reflector geometry were not considered. Also, the GENEPI-3C accelerator was not modelled. Instead, a 14-MeV point source was placed in vacuum at the core center. For the Am-Be source, the average source energy of 5 MeV was used. Finally, the FCs were not modelled at all. Instead, use was made of the next-event estimator MCNP tally F5 (point detector) to estimate the fission rates of the FC deposits, at the center of each detector location.

One MCNP input file was created for each CR height selected for the MSM experiments (from 0 to 600 mm by step of 60 mm around the reference CR position of the SC1↓ configuration and by step of 50 mm around the one of CR0↓). Then, prior to calculating the four terms of formula (15), the reactivity scale of the MCNP models of VENUS-F configurations had to be adjusted so that the calculated reactivities of CR0↓ and SC1↓ be approximately equal to their measured values of –136 pcm and –3824 pcm, respectively. This allowed an overall consistency between experimental results concerning the configurations used as references and the subsequent calculations. This was achieved by multiplying the average number of neutrons per fission ν used inside the MCNP code by a factor of 1.001071. This slightly modified value of ν was then used for calculating the reactivity of all the other configuration variants obtained by changing the CR heights.

3.3 Results of MSM factor calculations

Figure 4 shows the evolution of MSM factors as a function of the new height of the CRs after moving them away from their position associated with the reference configuration SC1↓ (479 mm). Error bars were calculated using the quadratic sum of the uncertainties on the four terms of formula (15). The relative uncertainty is basically
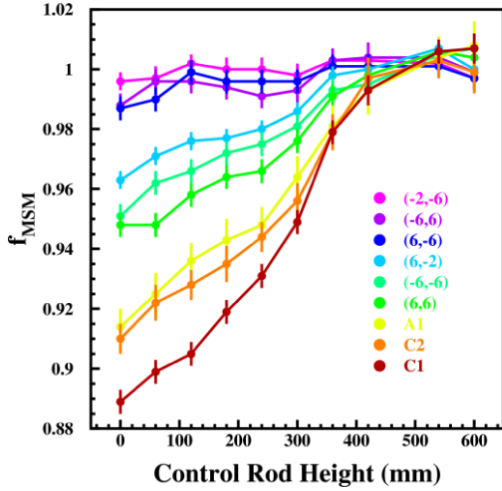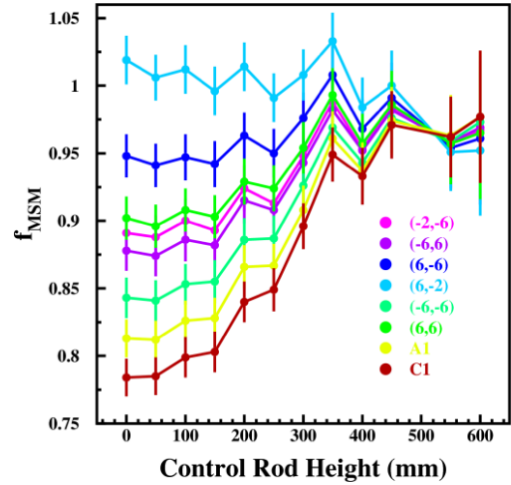
Fig. 4. MSM factors as a function of FC position and CR height using SC1↓ as reference.

Fig. 5. MSM factors as a function of FC position and CR height using CR0↓ as reference.

dominated by that on the fission reaction rates for which a precision of less than 1% could be achieved in a reasonable computing time.

The first observation to be made is that, as expected, regardless of the detector position, there is roughly no MSM correction to consider when the CRs do not move much around 479 mm. However, as the amplitude of the CR motion and thus the dissimilarity between the neutron flux shapes increases, the MSM factors tend to deviate more and more from 1. Regarding the evolution of the MSM factors, the detectors seem to fall into three or four different groups. In the first group (positions (–2,–6), (–6,6) and (6,–6)), the MSM factors do not deviate much from 1, even for the largest CR motion. This can be explained by the fact that those three detectors are rather far from the source and far from the CRs and therefore rather protected from the modifications of the flux shape caused by the CR motion. It is less and less the case as we move from the first group to the second one (detectors in (6,–2), (–6,–6), (6,6)) and then to the third one (detectors in A1, C2 and C1).

In conclusion, in the case of the SC1↓ configuration, it seems possible to estimate the CR worth using a simple ASM approach without any calculated correction factor as long as the detectors are carefully selected.

Figures 5 and 6 deal in the same way with the MSM factors for the CRs moving when the reactor is almost at critical, that is when the reference configuration used for the MSM calculation is CR0↓ (CR height at 515 mm). The color code for detector positions is identical to that of Figure 4. First of all, it is worth mentioning that the statistical error bars are significantly larger than those shown in Figure 4. Indeed, since CR0↓ is almost critical, the reactivity values are rather close to zero and the relative uncertainties on the calculated reactivity tend to be much larger than those calculated for the configurations based on SC1↓. Furthermore, some MCNP fixed-source calculations needed for estimating detector fission rates can become very computer-time-consuming as the CR height increases, and hence as the multiplication factor k_{eff} becomes very close to 1. To quantify this evolution, one can make use of the

Figure of Merit (FOM) [10] which is defined as:

$$FOM = \frac{1}{R^2 T}$$

where T is the computing time and R the relative statistical uncertainty on the quantity of interest (here the detector reaction rates). For instance, between fixed-source calculations performed at 0 mm and at 600 mm, the Figure of Merit drops by a factor of ∼300.

As in the case where the reference configuration is SC1↓, the MSM factors tend to deviate more and more from 1 as the amplitude of the CR motion increases, as expected by the associated larger perturbation of the reference flux. However, compared to Figure 4, two main differences are visible in Figures 5 and 6.

The most striking one concerns the FC located in C2 (Fig. 6). The behavior of the associated MSM factor is so different from the others that it had to be shown in a separate figure. This is due to the extremely short distance from the Am-Be source (13.5 cm) combined to the high energy
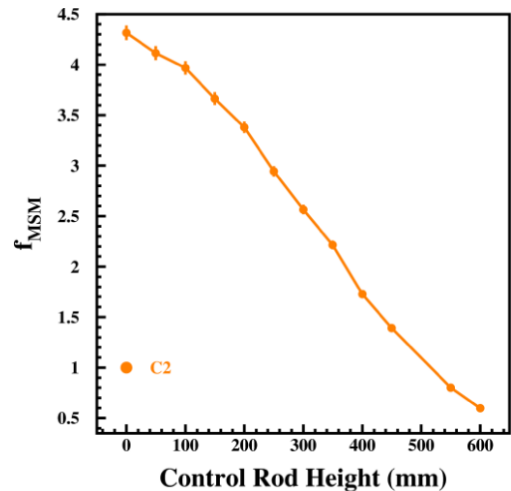
Fig. 6. MSM factors associated with detector CFUL01-673 in C2 as a function of CR height using CR0↓ as reference.

threshold of ^{238}U ($\sim$1 MeV) which represents 99.965% of the CFUL01-673 deposit mass. Thus, on the one hand, CFUL01-673 is proportionally much more sensitive than the other FCs to the fast neutrons originating directly from the Am-Be source and, on the other hand, it is much less sensitive to the regular neutron multiplication in the core.

When looking at Figure 5, it also appears that the clear division of detectors in groups proposed for Figure 4 does not hold any longer. Although some detectors very close to one CR exhibit strong correction factors, such as RS-10075, it is not the case for the FC located in (6,–2). On the other hand, the detector located in (–6,6) is very far away from the CRs and from the Am-Be source and still, its MSM correction factor is far from remaining close to 1 when the CRs are moved.

To understand why these MSM factors exhibit a much more complex behavior than in the case of those calculated for SC1↓ and its variants, it is worth using formula (12). The latter relates the MSM factors to the ratio of the source importance in the configuration of interest to that in the reference configuration. The source importance can be easily calculated by combining the results of MCNP kcode and fixed-source calculations.

As already mentioned herein above, the Am-Be source importance is much smaller ($\sim$0.3) than the GENEPI-3C one ($\sim$2.5), mainly because of the difference in the source location. Hence, it is more fruitful to compare the source importance variations (compared to the value of φ^* taken arbitrarily at 0 mm) as a function of CR height for the variants of the configurations based on CR0↓ and for those based on SC1↓. Results (in %) are shown in Figure 7.

The difference in behavior of the source importance between the two sets of reactor configurations is striking. On one hand, in the case of SC1↓-derived configurations (in red), the variation of source importance as the CR height changes, if any, is very small. On the other hand, when considering CR0↓-derived configurations (in black), the source importance appears to increase significantly as the CRs are raised. This is obviously due to the very short distance between the Am-Be source and one of the two CRs,

which makes the source multiplication very sensitive to the motion of the neutron absorbent (while the external neutron source was at the core center in the case of the variants based on SC1↓). Hence, whereas the MSM factor evolution seems to be mainly explained by the flux shape modification occurring around the CRs when they are moved from the 479 mm position for SC1↓, the MSM factor variation with the CR motion around CR0↓ seems to be more complex. In this case, the source importance and the flux shape are both modified.

In short, the behavior of the MSM correction factors as a function of the detector position appears to be much more complex when using the Am-Be source instead of the external neutron source provided by means of the GENEPI-3C accelerator. This difference is likely to originate from the off-centered position, close to one CR, of the Am-Be source.

4 Application to MSM experiments

It is beyond the scope of this paper to apply the MSM factors calculations presented herein to the numerous measurements performed at the VENUS-F reactor. Instead we present hereinafter a few results which illustrate the most the performances of the MSM method with Monte Carlo simulations.

4.1 MSM experiments with the SC1↓ configuration

As shown in Figure 4, the farther from that of the reference configuration (479 mm) the CR height is, the larger the MSM correction factor is. This is the reason why the results corresponding to the CR heights settled at 0 mm have been selected and are shown in Table 2.

First the ASM reactivity ρ_{ASM} of the VENUS-F reactor when the CRs are positioned at 0 mm was calculated from each detector count rate $\tilde{R}$ as follows:

$$\rho_{ASM}(0\,\text{mm}) = \frac{\tilde{R}(479\,\text{mm})}{\tilde{R}(0\,\text{mm})}\rho(SC1\downarrow) \qquad (16)$$
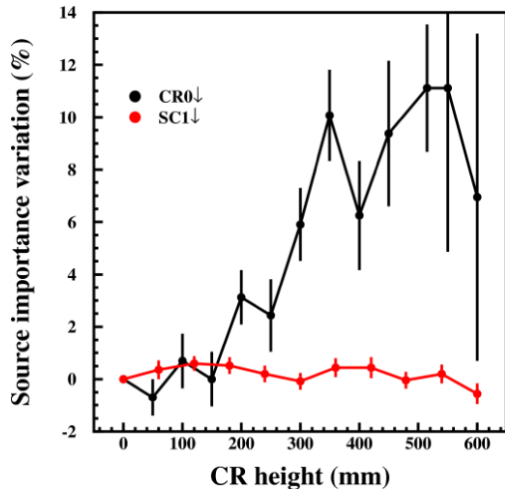

Fig. 7. Source importance variations as a function of the CR height.

Table 2. ASM reactivity, MSM factor and MSM reactivity when CRs are lowered to 0 mm (from SC1↓) for each of the nine FCs used.

Name	Location	ρ_{ASM}(pcm)	f_{MSM}	ρ_{MSM}(pcm)
CFUL01-658	A1	–4884(123)	0.914(6)	–4464(116)
CFUL01-659	(–6,6)	–4573(115)	0.988(4)	–4518(115)
CFUL01-653	C2	–4874(122)	0.910(5)	–4436(114)
RS-10071	(6,–6)	–4597(116)	0.987(4)	–4537(115)
RS-10072	(6,6)	–4792(120)	0.948(4)	–4543(116)
RS-10074	(–6,–6)	–4772(120)	0.951(4)	–4539(116)
RS-10075	C1	–5090(128)	0.889(4)	–4525(115)
CFUM21-667	(6,–2)	–4718(119)	0.963(3)	–4544(115)
CFUM21-668	(–2,–6)	–4576(115)	0.996(3)	–4558(115)

where $\rho(SC1\downarrow)$ is equal to –3824 $\pm$ 96 pcm. Since the neutron external source was created by means of the GENEPI-3C accelerator, a specific normalization had to be applied to the detector count rates, $\tilde{R}(479\text{ mm})$ and $\tilde{R}(0\text{ mm})$, measured respectively in the reference configuration (SC1↓, CRs at 479 mm) and in the other selected configuration (SC1↓, CRs at 0 mm). In order to take into account the fluctuations of the neutron production in the tritiated target, the total numbers of counts in the detectors were normalized per alpha particle detected in the dedicated Si detector, instead of per second.

As can be seen in Table 2, the ASM reactivity varies by about 550 pcm, depending on the detector considered. Some spatial effects are definitely at work. Now, applying the MSM factors calculated with MCNP simulations which are shown in Figure 4 and also listed in Table 2, the MSM reactivity can be built for each detector as follows:

$$\rho_{MSM}(0\text{mm}) = f_{MSM} \times \rho_{ASM}(0\text{mm}). \quad (17)$$

After multiplication by the MSM factors, the dispersion of the detector reactivity values is successfully reduced to ~120 pcm.

To finish with the SC1↓-related experiments, we make use of one interesting result of the MSM factor calculations. In Figure 4, one can see that the MSM factors for three FCs (positions (−2,−6), (−6,6) and (6,−6)) are very close to 1. This suggests that the simple ASM method applied to these three detectors might give the same results as the MSM method does. The comparison of the reactivity results of the ASM method applied to the three detectors with the MSM ones using the full set of FCs, as a function of CR height, is presented in Figure 8. For each CR position and each method, the reactivity was calculated as the weighted mean of the values given by the two different sets of selected detectors (3 and 9 FCS for the ASM and MSM methods, respectively):

$$\langle\rho\rangle = \frac{\sum_i \rho_i/s_i^2}{\sum_i 1/s_i^2} \quad (18)$$
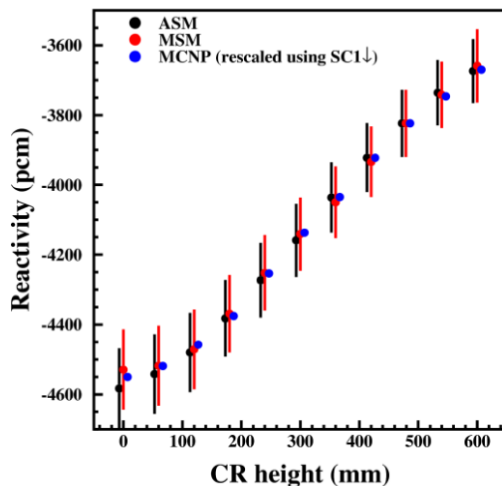
Fig. 8. Reactivity of VENUS-F as a function of CR height when CRs are moved together from 0 to 600 mm, around the SC1↓ configuration.

where s_i is the uncertainty on the reactivity given by detector i. The uncertainty associated with the average reactivity was conservatively calculated by assuming that correlation was at maximum between all the detectors considered:

$$s = \sqrt{\frac{\sum_i \sum_j 1/(s_i s_j)}{\left(\sum_i 1/s_i^2\right)^2}}. \quad (19)$$

As can be seen in Figure 8, the agreement between the two methods is excellent over the whole range of CR heights. Not only it validates the use of MSM factor calculations to choose the right detector subset to use an ASM approach but it also shows that the measurements made with all the other detectors can be well corrected. It is also remarkable that, once the kcode MCNP reactivity values are properly scaled by adjusting the calculated reactivity of SC1↓ to the measured one, they (in blue) also are in very good agreement with the experimental results over the whole range of CR height. In addition, it is worthwhile to mention that some dynamical reactivity measurements carried out in pulsed neutron source experiments [12] or in experiments with programmed interruptions of a continuous beam [13], with the CR heights at 0, 240, 479 and 600 mm, gave results consistent with those presented here.

4.2 MSM experiments with the CR0↓ configuration

Since, as in the case of the SC1↓ experiments, the MSM corrections to the ASM reactivity values are expected to be the strongest at 0 mm for the CR0↓ experiments, the results obtained for this height are gathered in Table 3. The spread observed among the ASM reactivity values is much more dramatic (~850 pcm) because of the presence of the CFUL01-673 FC with ^{238}U as main deposit (Sect. 3.3). However, once the MSM factors have been applied, the dispersion of the results drops down to ~80 pcm. Even though the CFUL01-673 final reactivity value seems to be

Table 3. ASM reactivity, MSM factor and MSM reactivity when CRs are lowered to 0 mm (from CR0↓) for each of the nine FCs used.

Name	Location	ρ_{ASM} (pcm)	f_{MSM}	ρ_{MSM} (pcm)
CFUL01-653	A1	−1013(13)	0.813(14)	−824(18)
CFUL01-659	(−6,6)	−950(12)	0.878(15)	−834(18)
CFUL01-673	C2	−210(5)	4.316(72)	−905(27)
RS-10071	(6,−6)	−876(12)	0.948(16)	−830(18)
RS-10072	(6,6)	−924(13)	0.902(16)	−833(19)
RS-10074	(−6,−6)	−987(14)	0.843(15)	−832(19)
RS-10075	C1	−1058(15)	0.784(14)	−829(19)
CFUM21-667	(6,−2)	−858(21)	1.019(18)	−874(27)
CFUM21-668	(−2,−6)	−937(23)	0.891(15)	−835(25)

slightly overestimated (in absolute value), one has to keep in mind that the MSM correction is a tour-de-force for this detector considering the amplitude of the correction to be applied.

5 Conclusions

The so-called Modified Source Multiplication Method (MSM) technique consists in determining the unknown reactivity of a reactor configuration by comparing detector count rates driven by an external neutron source in the configuration of interest with those obtained in another subcritical configuration whose reactivity is already known (reference configuration). This method can be used as an alternative method to the asymptotic period measurement for determining control rod worth.

This paper focused on the use of the Monte Carlo neutron transport code MCNP to calculate position-dependent MSM correction factors needed to account for the flux shape differences between the reference reactor configuration and the configuration whose reactivity is to be measured.

A comparison was made between the MSM factors obtained for a set of nine detectors spread in the lead-moderated fast neutron reactor VENUS-F, for a largely sub-critical configuration and a near-critical one. It was found that the MSM factors exhibited some common trends but also that the behavior of these factors was much more difficult to explain simply in the near-critical case because of the specific location of the external neutron source.

However, in both cases, the MSM factors calculated with the MCNP Monte Carlo code were successfully applied to ASM reactivity values obtained experimentally: the reactivity spread among the detectors was strongly reduced by the MSM correction.

In conclusion, the MSM method seems to be a technique well adapted to measure control rod worth over a large range of subcriticality levels. The required MSM factors can be easily calculated using a Monte Carlo neutron transport code, although the computing time can become very large when the studied reactor configurations are very close to criticality. Consequently, appropriate variance reduction techniques remain to be investigated.

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