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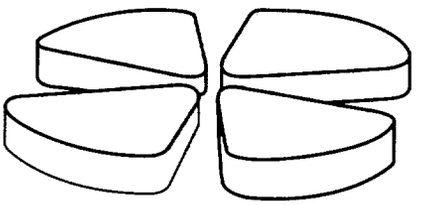

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Abstract:
The critical behavior of the fragment production is studied within a Lattice Gas Model in the canonical ensemble. Finite size effects on the liquid-gas phase transition are analyzed by a direct calculation of the partition function, and it is shown that phase coexistence and phase transition are relevant concepts even for systems of a few tens of particles. Critical exponents are extracted from the behavior of the fragment production yield as a function of temperature by means of a finite size scaling. The result is that in a finite system well defined critical signals can be found at supercritical (Kertész line) as well as subcritical densities.

Introduction

It is generally believed that heavy ion collisions may provide an experimental way to explore the properties of the nuclear matter under different thermodynamical conditions. Then, an important experimental issue is to extract information on the nuclear equation of state (EOS) from reaction data. In particular, considering the analogy between the nucleon-nucleon force and a Van der Waals interaction it is expected that the EOS presents the characteristic a liquid-gas phase transition. In the recent years, different indications of such a transition have been accumulated. On one side, studying the decay of hot nuclear systems it was observed a sudden opening of the multifragmentation [1, 2, 3] and vaporization [4] channels associated with the disappearance of a large residual nucleus. These behaviors were interpreted as signatures of the transition from the survival of a liquid phase at low temperature towards the massive production of a dilute gas system at high excitation energy through the apparition of a phase mixture in the intermediate energy domain [5, 6]. Moreover, this direct observation of various phases has been completed by the extraction of thermodynamical signals of first order phase transition. Indeed, it has been reported an anomaly of the evolution of the observed temperature as a function of the excitation energy (the so-called caloric curve) [7] which

shows a structure similar to a first order phase transition in the framework of statistical equilibrium models [8, 6].Moreover, statistical equilibrium models, extremely successful in reproducing multifragmentation patterns, systematically suggest low freeze out densities [9, 6] which are in complete agreement with the idea that the transition occurs in the middle of the coexistence region.

On the other hand, the observation of critical behaviors like power laws in the charge distribution of the multifragmenting system [10] has been interpreted as an evidence of a second order phase transition as expected in a single specific thermodynamical point, the critical point. Campi and Krivine have recently pointed out [11] that a critical behavior in the clustering properties of a multifragmenting system can be observed also at high densities (supercritical) along the Kertész line [12] thus suggesting the picture of an "early" multifragmentation when the system is dense. This idea is supported by molecular dynamics simulations where however thermodynamical equilibrium is clearly not reached [13].

The presence at the same time of first and second order transition signals is still a puzzle and the density at which the partitioning of the system occurs is still an open question. The main result of this paper is that a critical behavior in fragment observables can be consistent with the thermodynamics of phase coexistence and the occurrence of a low freeze out density due to finite size effects. In order to reach this conclusion we will study an exactly solvable model for second and first order phase transition, namely the Lattice Gas Model of Lee and Yang [14]. Of course this is a very simplified model which can be interpreted as a model for a classical fluid with a Van der Waals type of equation of state. Therefore, it should not be interpreted as a reasonable parametrization of the nuclear EOS but as a model case to investigate phase transitions and critical properties. The pertinence of the reached conclusions for the nuclear physics problematic relies on the fact that critical behaviors correspond to universal properties which do not depends on the details of the considered model.

Thermodynamics of the Lattice Gas Model

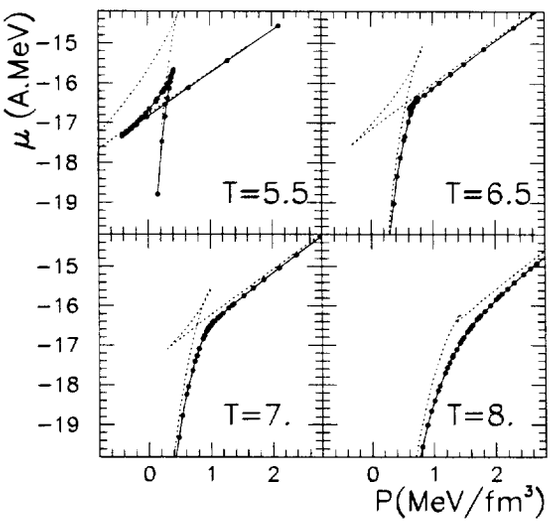

Figure 1. Chemical potential as a function of pressure at different temperatures for a cubic lattice of size $L = 6$ calculated from the exact partition sum (circles) and with the mean field approximation (dotted line).

The Lattice Gas Model where the grancanonical partition function of a gas with one type of atoms is mapped into the canonical ensemble of an Ising model for spin 1/2 particles in presence of an external magnetic field, has successfully described the liquid-gas phase transition for atomic systems. This same model has already been applied to nuclear physics in the grancanonical ensemble [15, 11], with an approximate sampling [16] of the canonical ensemble [17], and in the mean field approximation [18].

In our implementation the N sites of a lattice are characterized by an occupation number τ which is defined as $\tau = 0$ for a vacancy, and $\tau = 1$ for a nucleon. Particles occupying nearest neighboring sites interact with a coupling constant $\epsilon = -5.5$ MeV which is fixed such as to reproduce the saturation properties of nuclear matter. The Hamiltonian is given by

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m}\tau_i + \sum_{i \neq j} \epsilon \tau_i \tau_j \tag{1}$$

In all the calculations shown below the numerical realization of the model is a three-dimensional cubic lattice with periodic boundary conditions characterized by a size L, a number of particles A (or equivalently a density $\rho/\rho_0 = A/L^3$) and a temperature T. Statistical averages are taken over events obtained with a standard Metropolis sampling of the lattice occupations in the canonical ensemble.

The thermodynamics of the model is calculated from a direct evaluation of the partition sum Z via an iterative procedure (analogous to the one used in [19]). At a temperature T, the number of sampled realizations of the system with an energy state E as given by the Boltzmann probability is

$$N(E) = N \frac{1}{Z_T} W(E) e^{-E/T} \tag{2}$$

where $W(E)$ is the degeneracy of the state and N the total number of sampled states which is taken to be independent of the temperature. From the comparison of the occupation probabilities at two different temperatures T_1, T_2 we can then define the partition sum as

$$Z_1 = Z_2 \frac{N_1(E)}{N_2(E)} e^{-E\left(\frac{1}{T_2} - \frac{1}{T_1}\right)} \equiv z_1(E) \tag{3}$$

which is valid for all the different energy bins. In order to profit from all the available data we can compute the partition as an average of the above relation over the various energy bins

$$Z_1 = \sum_E z_1(E) \sqrt{N_2(E) N_1(E)} \Big/ \sum_E \sqrt{N_2(E) N_1(E)} \tag{4}$$

Then, Z_1 is obtained iteratively with an initial normalization to the infinite temperature limit where the partition sum is analytical. We have checked that an inversion of eq.(2) for each energy bin leads to an a posteriori estimation of the temperature statistically consistent with the initial one [20].

The equations of state can then be obtained from numerical derivative of the logarithm of the partition sum. As an example, in figure 1 a few isotherms are shown in the chemical potential versus pressure plane for a lattice of size $L = 6$ (*i.e.* a number of particles varying from $A = 10$ to $A = 210$). At low temperature, one can clearly observe two branches characteristic of a gas phase at low pressure and of a liquid phase at high density. This coexistence of two phases disappears above a critical temperature leaving a single fluid isotherm. Making a systematic analysis of the crossing points of the two branches the coexistence zone can be evaluated very accurately leading to a

critical temperature $T_c = 6.7$ MeV and a critical exponent $\beta = 0.31$ for the temperature dependence of the order parameter

$$\rho - \rho_c = (T - T_c)^{\beta} \tag{5}$$

This value has to be compared with the mean field approximation [17] (shown as dotted line in figure 1) $\beta = 0.5, T_c = 8.1$ and with the expected value in the thermodynamical limit $\beta^{\infty} = 0.33, T_c^{\infty} = 6.16$. It is clear that the finite size of accessible nuclear systems does not imply a drastic deformation of thermodynamical parameters (the corresponding values for $L = 8$ are $T_c = 6.6$ and $\beta = 0.31$). However the speed of convergence towards the thermodynamical limit strongly depends on the observable studied [11].

Fragment Production and Finite Size Scaling

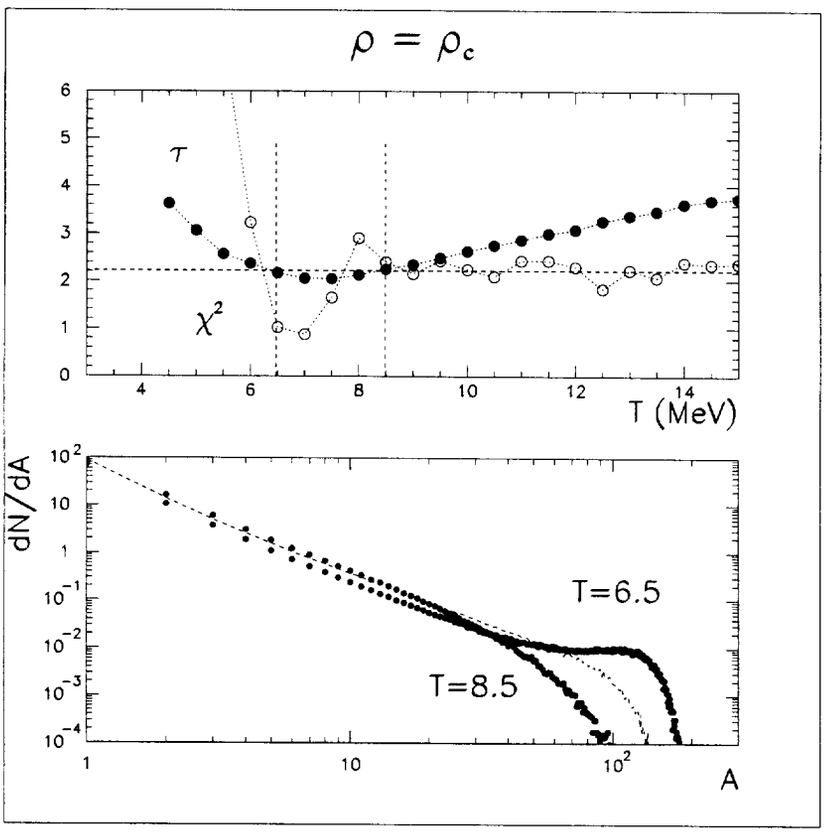

Figure 2. Top part: Exponent of the power law (filled circles) and corresponding χ^2 (open circles) of the size distribution as a function of temperature for A=256. Dashed lines: τ_{max} extracted from the maximum production yields (see text) and corresponding possible critical temperatures. Bottom part: size distribution for 3 temperatures around the critical one (6.5, 7.5 and 8.5).

The definition of clusters has been extensively discussed in the literature [11, 17]. The first idea is to group all the connected sites (Ising clusters). However, it is known since a long time that this is not the proper way to define clusters in the lattice gas model since it does not fullfil the requirement that the correlation length should diverge at the critical point (see [11] for a discussion of the literature). Using renormalization group arguments, Coniglio and Klein proposed to combines the above site percolation with an additional bond percolation algorithm using a temperature dependent bond breaking probability [21]:

$$p(T) = e^{-\epsilon/2T} \tag{6}$$

An alternative and almost equivalent way to define clusters is to break the bond between two nucleons as soon as the kinetic energy of their relative motion $\vec{p}_r^2/2\mu$ exceeds the binding energy ϵ. Remark that this bond breaking stage of the fragment definition can be understood as a secondary decay of the Ising Fragments. Typical fragment size distributions are displayed in figure 2.

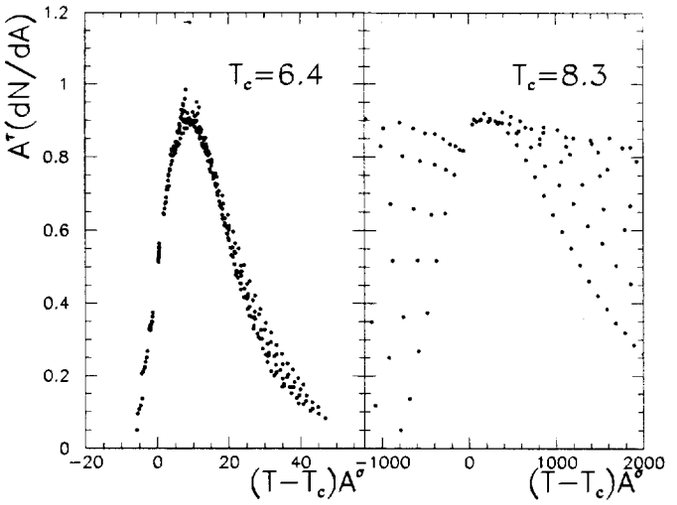

Figure 3. Scaling function for clusters of size ranging from 4<A<30 and temperature 2<T<20 obtained from eq.(7). In the left (right) part T_c was assumed to be the lower (upper) bound of the critical temperature region defined in figure 2.

With the above definition of clusters one expects that the thermodynamical critical point exhibits critical fragment size distributions and that in the proximity of the critical point the size distribution scales as [23, 22]

$$\frac{dN}{dA}(A,T) = A^{-\tau} f(A^{\sigma}(T - T_c)) \tag{7}$$

where f is a universal scaling function and τ, σ are critical exponents. Two particular values of the universal function are of particular interest:

- its value at the origin $f(0)$ since it is associated with the critical point

$$\frac{dN}{dA}(A,T_c) = A^{-\tau} f(0) \tag{8}$$

  which is the relation usually tested

- its maximum value $f(x_{\max}) = f_{\max}$ because then for each mass A it exists a temperature $T_{\max}(A) = T_c + x_{\max}A^{-\sigma}$ for which the production is maximum

$$\left(\frac{dN}{dA}\right)_{\max}(A) = \frac{dN}{dA}(A, T_{\max}(A)) = A^{-\tau} f_{\max} \tag{9}$$

Therefore a method to extract critical exponents consists in plotting the maximum production yield $(dN/dA)_{max}$ of a species of size A as a function of A; one can see from eq.(9) that this quantity should behave as a power law of exponent τ_{max}. Then T_c can be obtained as the temperature at which the power law fit to the size distribution $dN/dA(A)$ gives an exponent $\tau = \tau_{max}$. On figure 2 one can see that in finite size

systems two temperatures fullfil this condition. In order to decide which temperature should be associated with a critical behavior we have to extract the σ critical exponent and check the existence of a universal scaling function f by plotting $A^{\tau}\ dN/dA$ as a function of $A^{\sigma}(T-T_c)$ for all values of A and T. If all these different data points collapse on a single curve the validity of the universal scaling will be demonstrated. One can see on figure 3 that only the lowest critical temperature can be associated with a universal scaling behavior. In fact using the requirement that around the critical point the fragment production should exhibit a finite size scaling allows to define unambiguously the critical temperature within 0.5 MeV even for a system of a typical nuclear size.

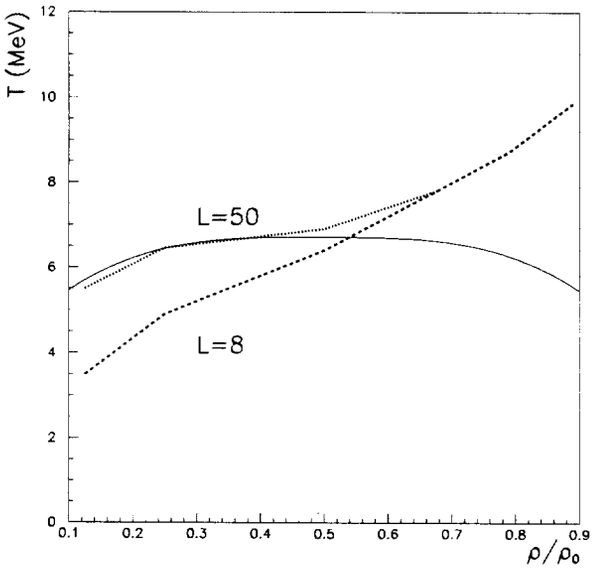

Figure 4. Full line: coexistence line from thermodynamics (see section 1); dashed line: critical curve from fragment size distributions for a cubic lattice of linear size $L = 8$; dotted line: as the dashed line, but $L = 50$

The result of the same analysis at different freeze out densities and lattice sizes is shown in figures 4 and 5 and typical fragment size distributions around the respective density dependent critical temperatures are shown in fig. 6. For all data points presented the quality of finite size scaling is comparable to the one observed at the critical density $\rho = 0.5\rho_0$.

A critical behavior is clearly seen at supercritical densities as well as at subcritical densities (dashed line in figure 4).However, the physical origin of critical behavior at subcritical densities lies on the finite size of the system. In the experimentally inaccessible limit of large nuclear systems, in the subcritical regime power laws are connected to the disappearance of the infinite cluster on the coexistence line (dotted line in figure 4) and not to the existence of critical fluctuations of all sizes, as it can be seen by directly looking at the fragment size distribution, figure 7. Indeed, for infinite (very large) system when the infinite cluster is disappearing we observe an exponential fragment size distribution rather then a power low. This explain why the critical exponents deviate from the expected value of their universality class (figure 5a, 5b).

It has already been observed that power laws in the size distribution are not characteristic of the critical point solely, but occur also at supercritical densities along the Kertész line [11] and at some subcritical densities at lower temperatures [17]. However, we have demonstrated in this paper the possibility to observe critical finite size scaling in small systems.

In this sense there is no contradiction between the scenario of fragmentation at

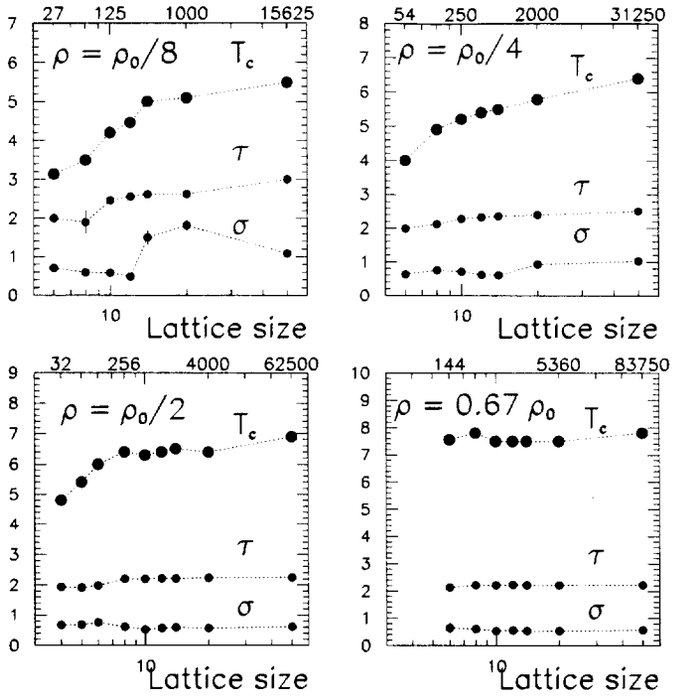

Figure 5. Critical parameters as a function of the linear size of the lattice (lower scale) or of the mass of the system (upper scale) at different freeze out densities.

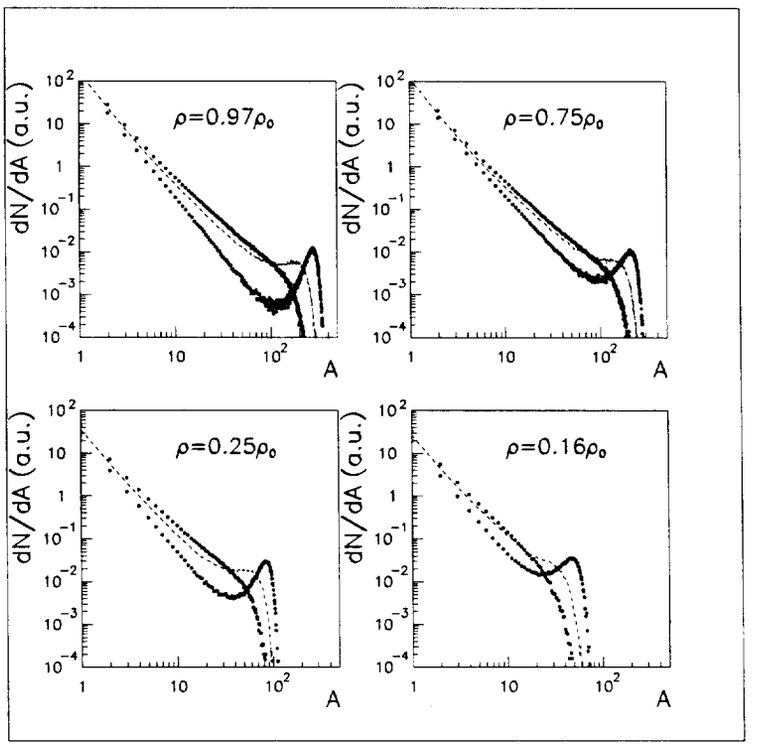

Figure 6. Fragment size distributions for a cubic lattice of linear size $L = 8$ at different densities and temperatures arround the critical one: T=9, 10.5 and 12 MeV for $\rho = 0.97\rho_0$; 7.5, 8.5 and 9.5 for $\rho = 0.75\rho_0$; 4.5, 5 and 5.5 for $\rho = 0.25\rho_0$ and finnally T=4, 4.5 and 5 MeV for $\rho = 0.16\rho_0$.

low density inside the coexistence or the spinodal region (first order phase transition) and the observation of critical signals characteristic of a second order phase transition. Moreover, if the critical temperature is extracted from data by verifying that finite size scaling is respected, critical parameters depend only very slightly on the mass for typical nuclear sources ranging from $A \approx 50$ to $A \approx 300$ and relevant thermodynamical informations can be extracted in spite of finite size effects (figure 5). However, the main problem is that the observation of a critical behavior is a signature that the system is lying on a critical line passing througjh the critical point and not that the system is actually at the critical density and temperature.

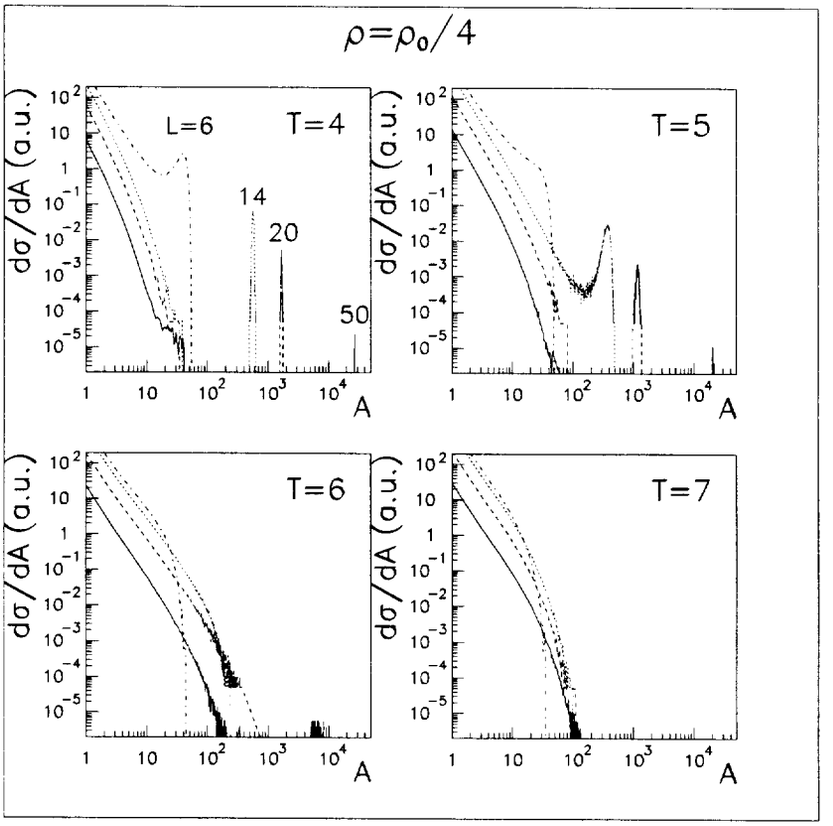

Figure 7. Fragment size distribution at $\rho/rho_0 = 0.25$ for different temperatures around the critical one and different linear size of the lattice L=6(dash-dotted lines), 14(dotted lines), 20(dashed lines) and 50(solid lines).

Conclusions

In this contribution we have looked for relevant signals of the nuclear liquid-gas phase transition within a canonical Lattice Gas model. An exact calculation of the canonical partition function has allowed us to compute the critical temperature within 0.1 MeV even for finite systems. We have shown that this variable is only very slightly perturbed by finite size effects. The critical temperature is also accessible from the fragment production yield, as well as the universal critical exponents of the transition. A full agreement between thermodynamical and fragment variables is obtained if the parameters are extracted from a systematic analysis of size distributions at different temperatures. In this case finite size scaling is remarkably verified over a wide range of sizes and temperatures at supercritical [11] as well as subcritical densities. At variance with the supercritical regime, critical behavior at subcritical densities is a side effect of

the finite size of the system which disappears in the infinite limit. However, the dependence with mass being smooth, the observation of critical signals from multifragmenting systems at low densities can provide useful informations on the characteristics and the parameters of the transition. The fact that the critical behavior is not confined to a single thermodynamical point, but can be seen along a whole critical line implies that the temperature and density of the multifragmenting source have to be inferred at the same time from experimental data. For a realistic application to fragmentation data however many important physical ingredients have to be added to the model. Surface effects have to be studied by replacing periodic boundary conditions with a confining Lagrange potential, the persistence of signals must be verified respect to the Coulomb interaction, and the possible influence on the results of the chosen thermodynamical ensemble (microcanonical versus canonical) has to be checked [20].

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