New ideas on gamma/phi3 measurements

A. Poluektov

New ideas in γ/ϕ_3 measurements

Anton Poluektov

Aix Marseille Univ, CNRS/IN2P3, CPPM, Marseille, France

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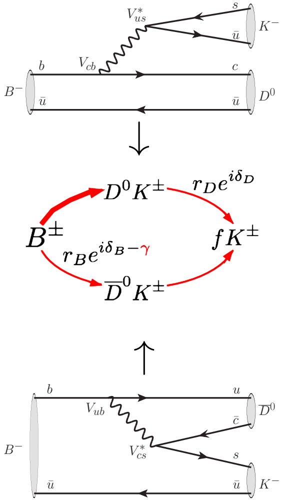
CKM 2021

Unitarity Triangle angle γ/ϕ_3

- Measured entirely in tree-level transitions in the interference of $b \to c$ and $b \to u$ diagrams.
- All hadronic parameters can be constrained from experiment
  $\Rightarrow$ theoretically very clean (uncertainty $< 10^{-7}$)
  [Brod, Zupan, JHEP 1401 (2014) 051]
- Combination of many different modes:
  - Time-integrated asymmetries in $B \to DK$, $B \to DK^*$, $B \to DK\pi$ with $D \to hh, hhhh$ ("ADS", "GLW")
  - Dalitz plot analyses of $D^0 \to K^0_S h^+ h^-$ from $B \to DK$, $B \to DK^*$ ("Dalitz" or "BPGGSZ")
  - Time-dependent analyses, e.g. $B^0_s \to D_s K$, $B^0 \to D\pi$



CP-asymmetric rates and experimental observables

Rate for $B \to DX$, $D \to f$ decay chain or its CP-conjugate:

$$\Gamma \propto r_D^2 + r_B^2 + 2\kappa r_D r_B \cos(\delta_B - \delta_D \pm \gamma)$$

Experimental observables:

r_B : ratio of $b \to u$ and $b \to c$ amplitudes

r_D : ratio of $D^0 \to f$ and $\overline{D}^0 \to f$ amplitudes ($\equiv 1$ for D_{CP})

δ_B and δ_D: corresponding strong phase differences

κ : coherence factor:

$\equiv 1$ for 2-body decays

< 1 if integrating over non-constant amplitude (binning) $\Rightarrow$ sensitivity diluted

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Take M $B \to DX$ modes, N $D \to f$ modes:

- $\sim (M \times N)$ measurements
- $\sim (M + N)$ unknowns (factorisation of B and D amplitudes!)

$\Rightarrow$ system of equations solvable w/o any theory input!

For multibody decays, can consider different kinematic regions as different decays, so γ measurement possible with only a single mode

Anything left to do?

Basically no theory uncertainty $\Rightarrow$ any experimental improvement pays off

- Inclusion of all modes with reasonable sensitivity
  - Correlations with hadronic "nuisance" parameters $\Rightarrow$ often more than just uncertainty of the average.
  - Better control of systematics, more robust measurement
- Statistically optimal usage of already available modes
- Understanding and control of uncertainties

Outline: some relatively recent ideas that did not yet materialise in real measurements

- Measurements with $D \to K^0\pi^+\pi^-$
  - Unbinned model-independent approach
  - Double Dalitz plot analysis with $B^0 \to DK^+\pi^-$
- Quantum-correlated $D^0\overline{D}^0$ pairs from $X(3872)$ decays
- Four-body D decays
- Decays of b-baryons

γ from $B^\pm \to DK^\pm$, $D \to K^0_S\pi^+\pi^-$

[Giri, Grossman, Soffer, Zupan, 2003; Bondar, 2002]

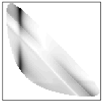
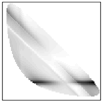
Allows for determination of γ without ambiguities.

Dalitz plot density: $d\sigma(m^2_+, m^2_-) \sim |A|^2 dm^2_+ dm^2_-$, where $m^2_\pm = m^2_{K_S\pi^\pm}$

Flavour D amplitude: $A_D(m^2_+, m^2_-)$

Amplitude of $D \to K^0_S\pi^+\pi^-$ from $B^+ \to DK^+$:

$$A_B(m^2_+, m^2_-) = A_D(m^2_+, m^2_-) + r_B e^{i\delta_B + i\gamma} A_D(m^2_-, m^2_+)$$

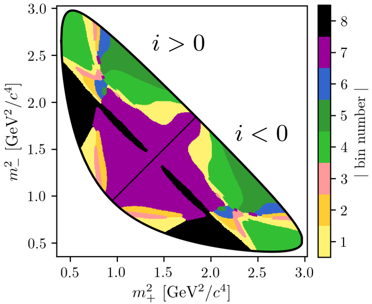
$=$  $+ r_B e^{i\delta_B + i\gamma}$ 

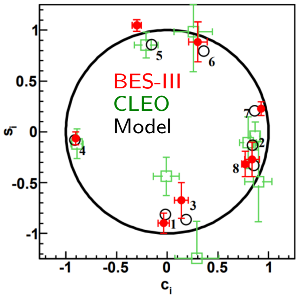
Need to know the strong phase difference between D^0 and $\overline{D}^0$

- **Model-dependent:** from $D \to K^0_S\pi^+\pi^-$ isobar model $\Rightarrow$ uncertainty
- **Model-independent:** from quantum-correlated $e^+e^- \to D^0\overline{D}^0$

Binned model-independent fit

[BESIII, PRL 124, 241802 (2020)]





Model-independent: system of equations for the bin yields:

$$N_i^{\pm} = h_{\pm}\left[K_i + (x_{\pm}^2 + y_{\pm}^2)K_{-i} + 2\sqrt{K_i K_{-i}}(x_{\pm}c_i + y_{\pm}s_i)\right]$$

- Physics parameters: $x_{\pm} = r_B \cos(\delta_B \pm \gamma)$, $y_{\pm} = r_B \sin(\delta_B \pm \gamma)$,
- Strong phase parameters: c_i, s_i from quantum correlations in $e^+e^- \to D\overline{D}$ decays.
- Flavour-specific bin yields: K_i

Coherence in bin i is determined by $s_i^2 + c_i^2$ ($\sim$ constant phase difference for better sensitivity)

Optimal binning chosen such as to maximise interference term $\Rightarrow \gamma$ precision

Phase coefficients c_i, s_i

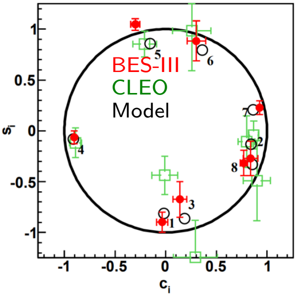
$c_i = \langle \cos \Delta\delta_D \rangle$, $s_i = \langle \sin \Delta\delta_D \rangle$ measured by CLEO (BESIII) in $e^+e^- \to D^0 \overline{D}^0$.
Density of correlated $D^{(\prime)} \to K^0_S \pi^+\pi^-$ Dalitz plots:

$$p_{DD}(m_+^2, m_-^2, m_+^{\prime\,2}, m_-^{\prime\,2}) \propto |A_D \overline{A}_D^\prime - \overline{A}_D A_D^\prime|^2$$

After binning:

$$M_{ij} \propto K_i K_{-j} + K_{-i} K_j - 2\sqrt{K_i K_{-j} K_{-i} K_j}(c_i c_j + s_i s_j)$$

which gives c_i, s_i in the fit.



c_i, s_i are aligned around a circle, well consistent with calculations from $D \to K^0_S \pi^+\pi^-$ model.

Do we really need 16 independent parameters to describe an (almost) circle in the phase?

Not really, but then need to go beyond simple binned approximation.

Carefully optimised binning has $\simeq 80\%$ power of the unbinned fit.

Can we do better? [AP, EPJC (2018) 78: 121]

Weight functions instead of **bins** in phase space $\mathbf{z} = (m^2_+, m^2_-)$:

$$\int_{\mathcal{D}_i} \dots d\mathbf{z} \quad \to \quad \int_{\mathcal{D}} \dots \times w_i(\mathbf{z})\, d\mathbf{z}$$

Treat decay densities as vectors in Hilbert space:

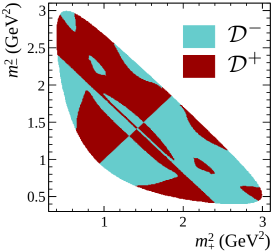
Projecting event density onto basis functions $w_i(\mathbf{z})$.

Works with scattered unbinned data (sum with weights).

E.g. **Fourier expansion** of strong phase difference:

$$w_{2n}(\mathbf{z}) = \cos\left(n\Delta\delta_D(\mathbf{z})\right);$$
$$w_{2n+1}(\mathbf{z}) = \sin\left(n\Delta\delta_D(\mathbf{z})\right)$$

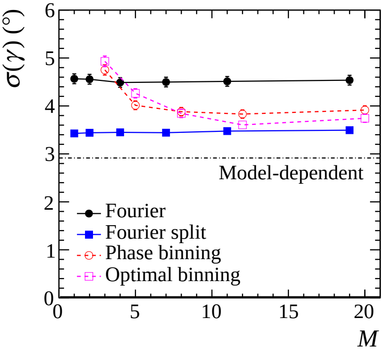
Additionally, can **split**
$\mathcal{D}^-$: $|A_D| < |\overline{A}_D|$ and $\mathcal{D}^+$: $|A_D| > |\overline{A}_D|$



[BESIII+LHCb inter-collaboration effort]

Toy MC sensitivity estimates

10^4 $D\overline{D}$ events, 10^4 $B^+ \to DK^+$ events, no background [AP, EPJC (2018) 78: 121]



Correct model for phase difference: optimal sensitivity already with 1st Fourier term

Splitting by amplitude: better precision than binned approaches.

Still does not reach model-dependent precision

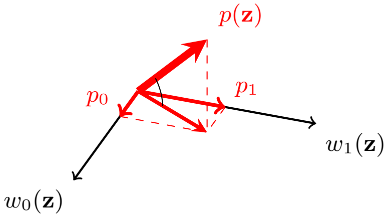
Further tuning possible, *e.g.* Legendre polynomials in amplitude ratio.

In reality, complications wrt. binned approach, *e.g.* correlations between observables.

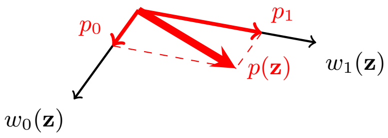
[BESIII+LHCb inter-collaboration effort]

[BESIII+LHCb inter-collaboration effort, very preliminary]

Optimal basis: limited set of basis functions $w_i(\mathbf{z})$ such that $p(\mathbf{z})$ fully lies in the subspace spanned by them (for any set of physics parameters).



$p(\mathbf{z})$ not fully contained in w_α span



$p(\mathbf{z})$ fully contained in w_α span

Density over $D \to K^0_S\pi^+\pi^-$ Dalitz plot from $B \to DK$ decays:

$$p_B(\mathbf{z}) = h_B\{p_D(\mathbf{z}) + r_B^2\bar{p}_D(\mathbf{z}) + 2[xC(\mathbf{z}) + yS(\mathbf{z})]\}$$

The density $p_B(\mathbf{z})$ is a **linear combination** of 4 functions:

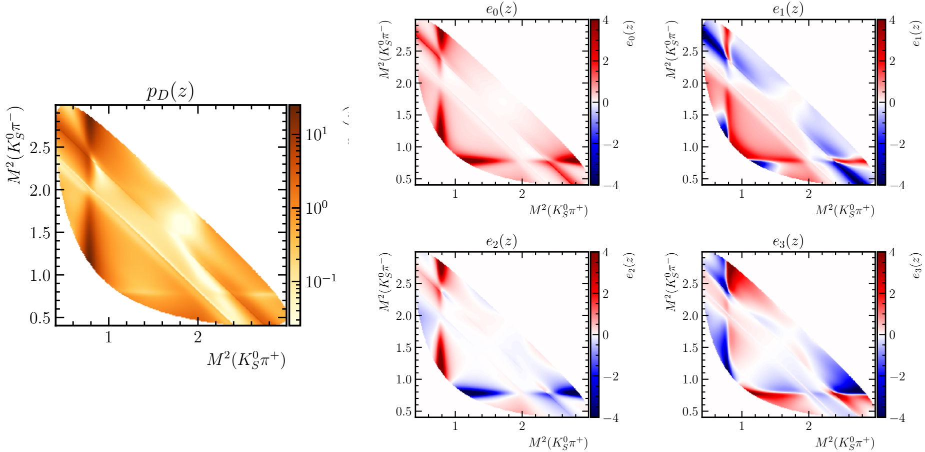
$$p_D(\mathbf{z}),\ \ \bar{p}_D(\mathbf{z}),\ \ C(\mathbf{z}) = \sqrt{p_D(\mathbf{z})\bar{p}_D(\mathbf{z})\sin\delta(\mathbf{z})},\ \ S(\mathbf{z}) = \sqrt{p_D(\mathbf{z})\bar{p}_D(\mathbf{z})\cos\delta(\mathbf{z})}$$

Use these 4 functions to create orthogonalised set of basis functions $\Rightarrow$

reach γ sensitivity equivalent to model-dependent fit

$D \to K_S^0 \pi^+ \pi^-$ optimal basis functions

Symmetrisation and orthogonalisation of $p_D(\mathbf{z})$, $\bar{p}_D(\mathbf{z})$, $C(\mathbf{z})$, $S(\mathbf{z})$



- Only 4 unknown strong phase parameters
- γ sensitivity expected to be **equal** to model-dependent fit
  - If the model used to define the basis is the true one, otherwise reduced
  - Still, the measurement will be unbiased
- Further improvements possible, *e.g.* > 4 functions for alternative models

Double Dalitz plot analysis of $B^0 \to DK^+\pi^-$, $D \to K^0_S\pi^+\pi^-$ decays

[T. Gershon, AP, PRD 81, 014025 (2010)], [D. Craik, T. Gershon, AP, PRD 97, 056002 (2018)]

- B^0 decays have larger interference term $r_B \sim 0.3$
- 3-body $B \to DK\pi$: amplitude and strong phase varies $\Rightarrow$ correlated B and D decay Dalitz plots.
- Applying the same model-independent binned technique to $B \to DK\pi$ decay

$$A_{\rm dbl\,Dlz} = \overline{A}_B \overline{A}_D + e^{i\gamma} A_B A_D \,,$$

After binning both Dalitz plots, system of equations:

$$\begin{aligned}\langle N_{\alpha i}\rangle \quad = \quad & h_{\rm dbl\,Dlz}\Big\{\overline{\kappa}_\alpha K_i + \kappa_\alpha K_{-i} \\ & + 2\sqrt{\kappa_\alpha K_i \overline{\kappa}_\alpha K_{-i}}\,[(\varkappa_\alpha c_i - \sigma_\alpha s_i)\cos\gamma - (\varkappa_\alpha s_i + \sigma_\alpha c_i)\sin\gamma]\Big\},\end{aligned}$$

Can be solved with three classes of events:

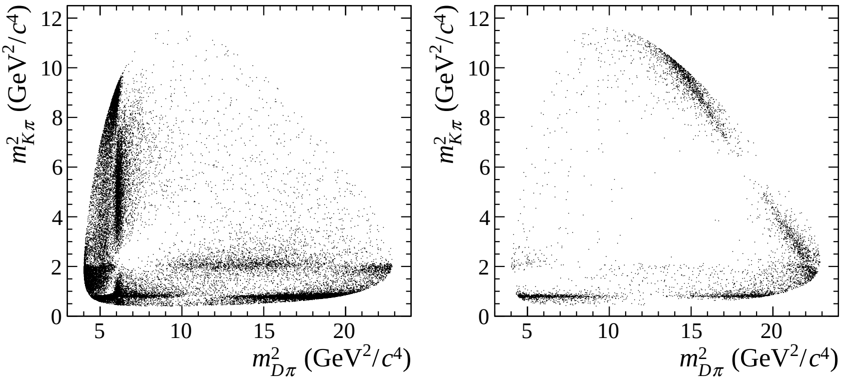
- $B \to DK\pi$, $D \to K^-\pi^+$ $(i = 1, c_1 = \cos\delta_{K\pi}, s_1 = \sin\delta_{K\pi}, K_1/K_{-1} = r^2_{K\pi})$
- $B \to DK\pi$, $D \to K^-K^+, \pi^-\pi^+$ $(i = 1,\ c_1 = +1, s_1 = 0, K_1 = K_{-1})$
- $B \to DK\pi$, $D \to K^0_S\pi^+\pi^-$

ADS-like mode contaminated by $B^0_s \to D^*K\pi$ decays at LHCb, study if the fit works after removing it (but can be added at Belle II)

[D. Craik, T. Gershon, AP, PRD 97, 056002 (2018)]

$B^0 \to D^0 K^+ \pi^-$ model from [PRD92, 012012(2015)]

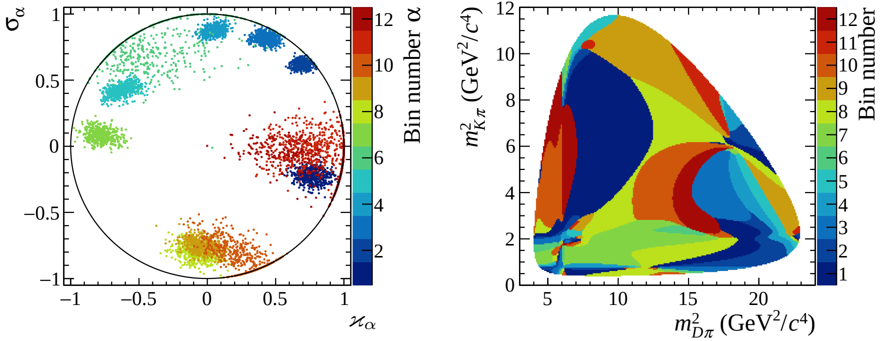
$B^0 \to \overline{D}^0 K^+ \pi^-$ model from γ analysis [PRD93, 112018(2016)],



$D \to K^0_S \pi^+ \pi^-$ model from Belle measurement [PRD81, 112002(2010)]

$B \to DK\pi$ binning

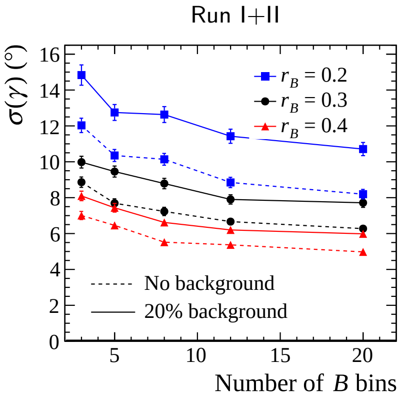
Optimal binning depends on $B \to DK\pi$ model, calculated by maximising interference term

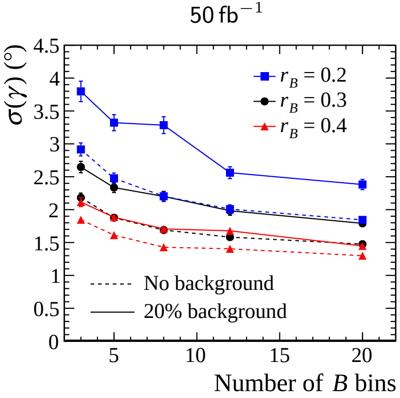


$\varkappa_\alpha$ and σ_α are $B \to DK\pi$ amplitude coeffs similar to c_i, s_i for $D \to K^0_S\pi^+\pi^-$.

Treated as free parameters in the fit.

Estimated LHCb sensitivity with this mode (current sample and after Upgrade 1).
Include 20% $B_s^0 \to D^* K^- \pi^+$ background for $D \to hh$ and $D \to K_S^0 \pi^+ \pi^-$.



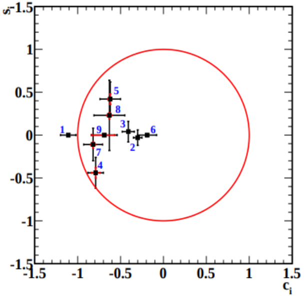


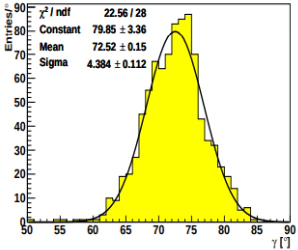
Background effect depends on r_B; reasonable precision even in worst case $r_B = 0.2$:
$\sigma(\gamma) \simeq 10^\circ$ in Run I+II and 2.5° with 50 fb^{-1}

4-body decays can be used in phase-space-integrated way

- Need to introduce coherence factor κ (in the case of ADS-like) or CP fraction $F_{\pm}$ (GLW-like mode). Measured model-independently in e^+e^- data.
- More optimal: bins in phase space (5D!). Need a model for binning optimisation.
- A few modes are analysed with CLEO-c data.

$D^0 \to K^0_S \pi^+\pi^-\pi^0$ [Resmi P.K. et al., JHEP 01 (2018) 082]





No amplitude model yet, bins defined around known resonances
Belle analysis [Seema Bhanipati's talk]; Belle II sensitivity estimate

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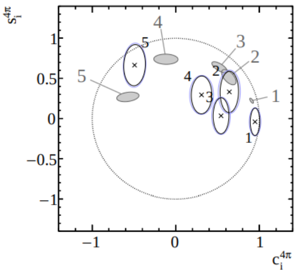
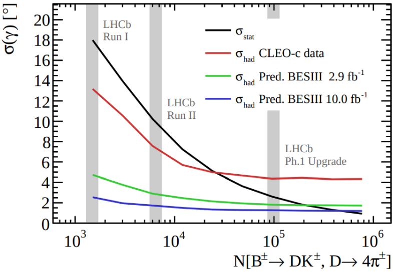
- A few modes are analysed with CLEO-c data.

$D^0 \to 4\pi$

[S. Harnew et al., JHEP 01 (2018) 144]

Optimal Binning





Amplitude model is fitted and several binning options determined
LHCb sensitivity estimates

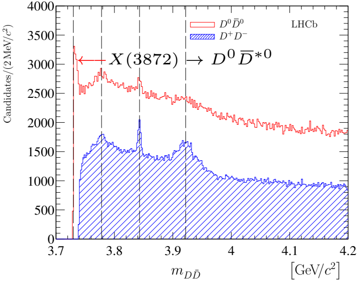
[P. Naik, arXiv:2102.07729]

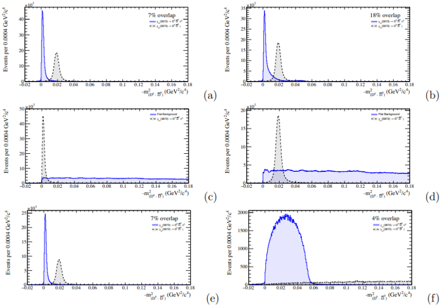
$X(3872) \to D^0\overline{D}^{*0}$ exactly at threshold

$X(3872)$ can serve as a source of quantum-correlated $D^0\overline{D}^0$ pairs. $J^{PC} = 1^{++}$

- $X \to D^0\overline{D}^0\pi^0$: $C = 1$
- $X \to D^0\overline{D}^0\gamma$: $C = -1$

[LHCb, JHEP 1907 (2019) 035]





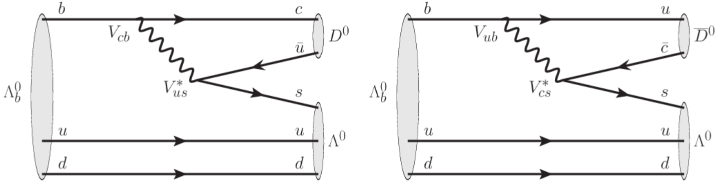
Visible at LHCb in prompt $D^0\overline{D}^0$ events.

Possible to distinguish $D^0\overline{D}^0\pi^0$ and $D^0\overline{D}^0\gamma$ without reconstructing soft π^0/γ

$\Lambda_b^0 \to DpK^-$ decays

Unique measurement for LHC: b-baryons. [Giri, Mohanta, Khanna, PRD 65 (2002) 073029]

γ-sensitive modes in the case of Λ_b^0:



$\Lambda_b^0 \to D\Lambda^0_{\to p\pi^-}$ mode:

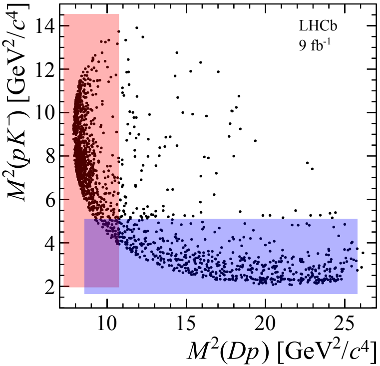
- S- and P-wave amplitudes with different strong parameters. Distinguish in $\Lambda^0 \to p\pi^-$ angular distribution
- At LHCb, affected by low efficiency to reconstruct long-lived Λ^0.

First try with excited, strongly decaying $\Lambda^{*0} \to pK^-$ instead.

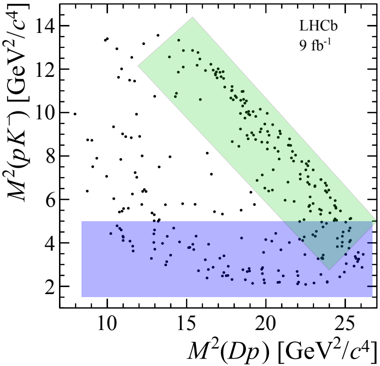
- Search for suppressed mode $\Lambda_b^0 \to DpK^-$ with $D \to K^+\pi^-$ (ADS-like)
- Measure CP asymmetry

[see presentation by Matteo Bartolini]

Favoured:
$\Lambda_c^{*+} \to D^0 p$ and $\Lambda^{*0} \to pK^-$



Suppressed
$D_s^{*-} \to \overline{D}^0 K^-$ and $\Lambda^{*0} \to pK^-$



$\Lambda_b^0 \to \Lambda_c^{*+} K^-$ $(b \to c)$ and $\Lambda_b^0 \to D_s^{*-} p$ $(b \to u)$ amplitudes are flavour-specific.

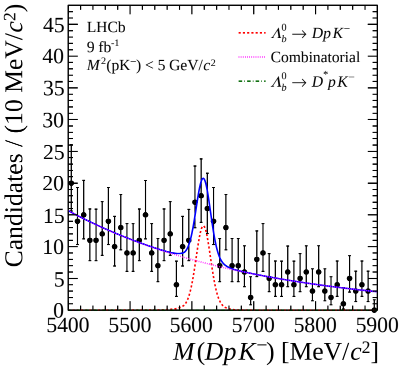
Taking only $\Lambda_b^0 \to D\Lambda^{*0}$ $(M^2(pK^-) < 5\,\mathrm{GeV}^2/c^4)$ should enhance CPV term

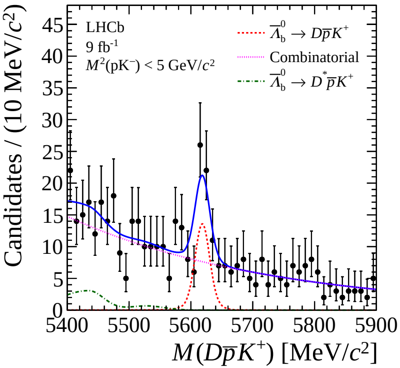
[LHCb, arXiv:2109.02621]

CPV in $\Lambda_b^0 \to DpK^-$

[see presentation by Matteo Bartolini]

CP asymmetry in the $\Lambda^{*0} \to pK^-$ region $(M^2(pK^-) < 5\,\mathrm{GeV}^2/c^4)$





$$R = 8.6 \pm 1.5\,(\text{stat.})^{+0.4}_{-0.3}\,(\text{syst.}),$$
$$A = 0.01 \pm 0.16\,(\text{stat.})^{+0.03}_{-0.02}\,(\text{syst.}).$$

No CPV today

[LHCb, arXiv:2109.02621]

γ from b-baryon decays: what's next?

Even if we measure nonzero CP asymmetry in $\Lambda_b^0 \to DpK^-$, can we extract γ?

- Λ_b^0 decays are more complex because of overlapping helicity states
  - Each Λ^{*0} helicity has, in general, its own strong phase
  - Polarisations of initial and final states (S- and P-wave amplitudes)
  - $\Rightarrow$ effectively, low and unknown coherence factor κ.
- $\Lambda_b^0 \to D\Lambda^0$ case with weak $\Lambda^0 \to p\pi^-$ decay:
  - Weak decay as a polarimeter, measure Λ^0 polarisation and resolve γ
  - See e.g. [Giri, Mohanta, Khanna, PRD 65 (2002) 073029]
- $\Lambda_b^0 \to D\Lambda^{*0}$ is different because $\Lambda^{*0} \to pK^-$ is strong (P-conserving)
  - Cannot determine Λ^* polarisation from angular distribution
  - Unfortunately, Λ_b^0 are not polarised in pp. Can we *make* them polarised?
  - Could exploit correlations of two b baryons [Yu. Grossman, private communication]. Mostly should be in $L = 1$. *Polarisation tagger?*

Summary

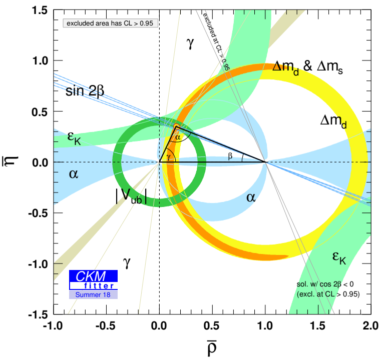
- γ measurements are not limited by theory uncertainties with the current precision
  - Effects such as charm mixing, K^0_S regeneration *etc.* can be controlled and are generally below 1°.
- Can be constrained by the combination of many different approaches $\Rightarrow$ more robust measurement
- Possibility to improve precision with already existing data
- More decay modes to be added in the future $\Rightarrow$ Hope the precision to exceed the one quoted in LHCb and Belle II performance papers.

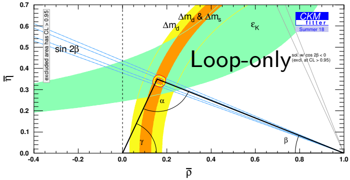
Backup

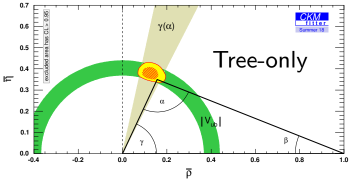
Cabibbo-Kobayashi-Maskawa matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

Sensitivity to BSM effects from the global consistency of various measurements





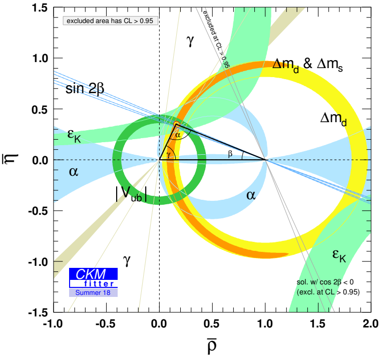


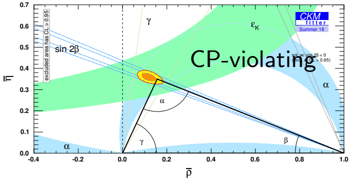
Unitarity Triangle measurements

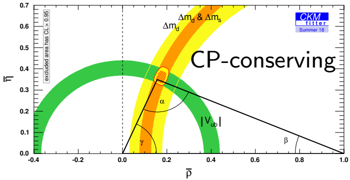
Cabibbo-Kobayashi-Maskawa matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

Sensitivity to BSM effects from the global consistency of various measurements







Charm data observables:

$$p_D(\mathbf{z}) = |A_D(\mathbf{z})|^2, \quad \bar{p}_D(\mathbf{z}) = |\overline{A}_D(\mathbf{z})|^2$$

$B^\pm \to DK^\pm$ **data observables**:

$$\bar{p}_B(\mathbf{z}) \propto p_D(\mathbf{z}) + r_B^2 \bar{p}_D(\mathbf{z}) + 2[x_+ C(\mathbf{z}) - y_+ S(\mathbf{z})]$$
$$p_B(\mathbf{z}) \propto \bar{p}_D(\mathbf{z}) + r_B^2 p_D(\mathbf{z}) + 2[x_+ C(\mathbf{z}) + y_+ S(\mathbf{z})]$$

Quantum-correlated $D^0\overline{D}^0$ data observables:

$$p_{DD}(\mathbf{z}_1, \mathbf{z}_2) \propto p_D(\mathbf{z}_1)\bar{p}_D(\mathbf{z}_2) + p_D(\mathbf{z}_2)\bar{p}_D(\mathbf{z}_1) - 2\left[C(\mathbf{z}_1)C(\mathbf{z}_2) + S(\mathbf{z}_1)S(\mathbf{z}_2)\right]$$

Unknowns:

$$C(\mathbf{z}) = \mathrm{Re}\left[A_D^*(\mathbf{z})\overline{A}_D(\mathbf{z})\right], \quad S(\mathbf{z}) = \mathrm{Im}\left[A_D^*(\mathbf{z})\overline{A}_D(\mathbf{z})\right].$$

We want to relate $p_D(\mathbf{z}), \bar{p}_B(\mathbf{z})$ and $p_{DD}(\mathbf{z}_1, \mathbf{z}_2)$ and eliminate $C(\mathbf{z}), S(\mathbf{z})$.
We need a way to do it with scattered experimental data.

Model-independent formalism with weighted integrals

Trick: replace all functions $f(\mathbf{z}) \to \int_{\mathcal{D}} f(\mathbf{z}) w_n(\mathbf{z}) d\mathbf{z}$

where $f(\mathbf{z}) = p_D(\mathbf{z}), \bar{p}_B(\mathbf{z}), C(\mathbf{z}), S(\mathbf{z})$.

$w_n(\mathbf{z})$, $1 \leq n \leq N$ is a family of certain weight functions.

Similarly, $p_{DD}(\mathbf{z}_1, \mathbf{z}_2) \to \int_{\mathcal{D}} p_{DD}(\mathbf{z}_1, \mathbf{z}_2) w_m(\mathbf{z}_1) w_n(\mathbf{z}_2) d\mathbf{z}_1 d\mathbf{z}_2$

All the equations will still hold, for any $w_n(\mathbf{z})$.

For scattered data, replace integrals by sums over individual events.

Binned approach is a particular case with

$$w_n(\mathbf{z}) = \begin{cases} 1 \text{ if } \mathbf{z} \in \mathcal{D}_n \\ 0 \text{ otherwise} \end{cases} \quad \text{for bins defined by } \mathcal{D}_n.$$

Altermative approach: **Fourier analysis of the modelled phase difference**

$$w_{2n}(\mathbf{z}) = \sin n\Phi(\mathbf{z}), \quad w_{2n+1} = \cos n\Phi(\mathbf{z})$$

where

$$\Phi(\mathbf{z}) = \arg A_D^{(\text{model})}(\mathbf{z}) - \arg \overline{A}_D^{(\text{model})}(\mathbf{z})$$

Fourier coefficients from scattered data

Calculation of Fourier coefficients from scattered data $\phi^{(i)}$:

$$a_n = \frac{1}{\pi}\sum_{i=1}^{N}\cos(n\phi^{(i)}), \quad b_n = \frac{1}{\pi}\sum_{i=1}^{N}\sin(n\phi^{(i)}),$$

For ML fit, also need covariance matrix (uncertainties and correlations) coming from the limited sample size. It can be calculated by applying Poisson bootstrapping: each term entering the sum is multiplied by a Poisson-distributed random number with mean of 1.

E.g. dispersion is calculated from central limit theorem:

$$\sigma^2(a_n) = \frac{1}{\pi}\sum_{i=1}^{N}\cos^2(n\phi^{(i)}), \quad \sigma^2(b_n) = \frac{1}{\pi}\sum_{i=1}^{N}\sin^2(n\phi^{(i)}),$$

This is a certain approximation, but seems to work well for $N > 100$ (pulls are compatible with 1).

Formalism: $B \to DK\pi$, $D \to K^0_S\pi^+\pi^-$

Two amplitudes, $A_D(m^2_{K^0_S\pi^+}, m^2_{K^0_S\pi^-})$ and $A_B(m^2_{D\pi^+}, m^2_{K^+\pi^-})$:

$$A_{\rm dbl\,Dlz} = \overline{A}_B\overline{A}_D + e^{i\gamma}A_BA_D\,, \qquad (1)$$

After $|\ldots|^2$ and binning both B and D Dalitz plots ($\alpha = 1\ldots\mathcal{M}$):

$$\begin{aligned}\langle N_{\alpha i}\rangle \quad = \quad & h_{\rm dbl\,Dlz}\Big\{\overline{\kappa}_\alpha K_i + \kappa_\alpha K_{-i} \qquad (2)\\ & + 2\sqrt{\kappa_\alpha K_i\overline{\kappa}_\alpha K_{-i}}\,[(\varkappa_\alpha c_i - \sigma_\alpha s_i)\cos\gamma - (\varkappa_\alpha s_i + \sigma_\alpha c_i)\sin\gamma]\Big\}\,,\end{aligned}$$

Strong phase terms for B amplitude:

$$\varkappa_\alpha = \frac{\int\limits_{\mathcal{D}_\alpha} |A_B||\overline{A}_B|\cos\delta_B\,d\mathcal{D}}{\sqrt{\int\limits_{\mathcal{D}_\alpha} |A_B|^2 d\mathcal{D} \int\limits_{\mathcal{D}_\alpha} |\overline{A}_B|^2 d\mathcal{D}}}\,, \qquad \sigma_\alpha = \frac{\int\limits_{\mathcal{D}_\alpha} |A_B||\overline{A}_B|\sin\delta_B\,d\mathcal{D}}{\sqrt{\int\limits_{\mathcal{D}_\alpha} |A_B|^2 d\mathcal{D} \int\limits_{\mathcal{D}_\alpha} |\overline{A}_B|^2 d\mathcal{D}}}\,. \qquad (3)$$

System of equations (5) for $B \to DK\pi$, $D \to K^0_S\pi^+\pi^-$:

$$\begin{aligned}\langle N_{\alpha i}\rangle \quad &= \quad h_{\mathrm{dbl\,Dlz}}\Big\{\overline{\kappa}_\alpha K_i + \kappa_\alpha K_{-i} \\ &\qquad + 2\sqrt{\kappa_\alpha K_i \overline{\kappa}_\alpha K_{-i}}\left[(\varkappa_\alpha c_i - \sigma_\alpha s_i)\cos\gamma - (\varkappa_\alpha s_i + \sigma_\alpha c_i)\sin\gamma\right]\Big\},\end{aligned}$$

And similar for c.c. decay ($\gamma \to -\gamma$). Enough constraints to measure γ.

Knowns and unknowns:

- K_i: from flavour-tagged $D \to K^0_S\pi^+\pi^-$
- c_i, s_i: from charm factory (or even free parameters)
- $\kappa_\alpha, \overline{\kappa}_\alpha$: from $B^0 \to D^0 K^+\pi^-$ (fav) and $B^0 \to \overline{D}^0 K^+\pi^-$ (sup) ($D^0 \to K^-\pi^+$)
- $\varkappa_\alpha, \sigma_\alpha$, γ: free parameters

Estimated yields

Run toys for expected yields after the end of Run II ($8\,\text{fb}^{-1}$) and $50\,\text{fb}^{-1}$, which correspond to $\times 4$ and $\times 65$ Run I yields (taking into account larger B CS at 13 TeV and $\times 2$ trigger efficiency in Run III).

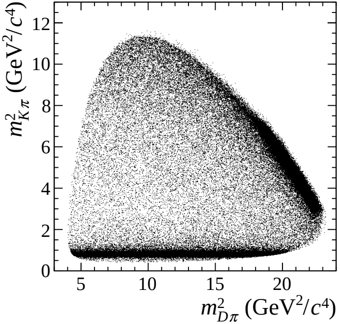
D decay mode	Run I	Run I+II	$50\,\text{fb}^{-1}$
$K^+\pi^-$	2 240	9 200	140 000
$K^-\pi^+$	220	900	14 000
K^+K^-	270	1 100	17 000
$\pi^+\pi^-$	130	540	8 500
$K^0_S\pi^+\pi^-$	420	1 700	27 000

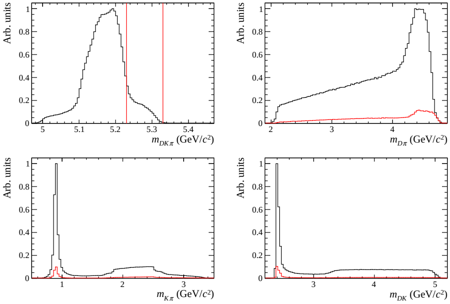
Run I yields are taken from corresponding analyses ($D \to K\pi$, KK, $\pi\pi$), for $D \to K^0_S\pi^+\pi^-$ they are extrapolated from $B \to DK^*$, $D \to K^0_S\pi^+\pi^-$ yield using the measured $B \to DK\pi$ model.

$B_s^0 \to D^* K^- \pi^+$ background

$B_s^0 \to D^* K^- \pi^+$ is unavoidable background for LHCb, dangerous since produces opposite-flavour D (looks like suppressed amplitude from slide 9).

Amplitude structure not studied, but can make an educated guess based on $B_s^0 \to D K \pi$ analysis and known $D^* K$ resonances. Simulated here:
$D^* K^*$, $D_{s1}(2536)^- \pi^+$, $D_{s2}^*(2573)^- \pi^+$, $D_{s1}^*(2700)^- \pi^+$, nonres $D^* K \pi$.





Cocktail simulated using [RapidSim] and [EvtGen] (incoherent, but correct helicity structure for non-scalars)