# Minimal Flavor Violation

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# Contents

# Introduction

The Standard Model (SM) is a very successful theory. Once its nineteen free parameters are measured, all sorts of reactions among particles or particle properties can be predicted and confronted with experiment. No significant deviation has ever been seen and subsequently confirmed. Impressively, the precision achieved both theoretically and experimentally permits to check the model at the quantum level, i.e., at one, two, or even three loops. Even when perturbation theory collapses, as for low energy observables sensitive to strong interaction effects, tools like effective theories or numerical simulations have been designed to obtain non-perturbative results. Though the precision is not always tremendous, the overall picture is here also of a systematic agreement with the SM.

## On the need for New Physics

So, why do we believe this model is not the end of the road? There are two kinds of hints. First, the SM appears incomplete. There must be some new degrees of freedom or dynamics beyond:

- **Neutrino masses:** The SM in its minimal version does not account for neutrino masses. This is rather easily remedied, and does not necessarily require a deep reappraisal of the model. However, their lightness may hint at the presence of a new high energy scale, the seesaw scale at around $10^9 - 10^{13}$ GeV [1], at which new degrees of freedom could exist.

- **Gravitons:** Another missing piece is gravity, under its yet to be discovered quantized form [2]. Lacking this, all we can say it that the range of validity of the SM cannot extend beyond the Planck scale $M_{Panck} = G^{-1/2} \approx 10^{19}$ GeV. In this sense, the renormalizability of the SM, no matter how nice and computationally convenient, may not be indispensable since there is in any case a natural high energy cut-off where the theory needs to be amended.

- **Dark matter / Dark energy**: The existence of dark matter is now an established fact [3]. From our current cosmological models, this "stuff" could be made of some not too heavy (nor too light) particles interacting very weakly with normal matter. Such a particle or set of particles cannot be any of those present in the SM. Dark energy, for its part, is far more mysterious. It is not even clear that a standard particle interpretation is adequate [4].

- **Baryogenesis:** The current universe seems very unbalanced. Matter, including baryons, totally dominates over antimatter [5]. The SM does not have enough $\mathcal{CP}$ violation, the required baryon number violation ($\mathcal{B}$), or the thermodynamic properties required [6] to create such a universe. There must be either new $\mathcal{CP}$ and $\mathcal{B}$ violating dynamics, or new degrees of freedom.

In addition to these missing pieces, the second kind of hints for new physics (NP) is to be found in the free parameters of the SM. First, obviously, it is always aesthetically nice to reduce the number of free parameters. This has driven most of the theoretical advances since Maxwell wrote his equations. But in addition, looking at their measured values, the SM parameters do not seem so free after all.

Instead, some patterns appear and beg for a deeper theoretical interpretation. The most striking features are:

- **Gauge coupling unification:** The independent coupling constants of the three SM gauge groups evolve with the energy, and appear to meet at around $10^{14}-10^{16}$ GeV [7,8]. It is tempting to interpret this as the signal that the SM gauge group originates from a spontaneously broken larger gauge group. Indeed, if this group is simple (i.e., not a factor group), then the Lie algebra commutation rules fix the couplings of all charged particles to be universal. This unique coupling constant then branches into our three couplings through their separate evolutions down from the GUT breaking scale. The simplest candidate group is $SU(5)$, and has the additional feature of predicting the fermionic charge quantization by unifying quarks and leptons into fundamental multiplets. Unfortunately, there are several unresolved issues in the simplest scenarios, and the final word on the GUT idea is not yet known.

- **Hierarchy puzzle:** The free parameters of the SM scalar potential have to be renormalized, as all the other free parameters. But, the peculiarity of these bare parameters is that their radiative corrections diverge quadratically. So, if one choose to regularize those divergences using a high energy cut off, say $M_{Planck}$ or $M_{GUT}$, then their corrections will be proportional to $M_{Planck}^2$ or $M_{GUT}^2$. Given that the renormalized (physical) values of the Higgs boson mass and its vacuum expectation values are both of the order of the electroweak scale, very delicate fine-tuning of the bare scalar parameters are required [9]. Of course, as long as one views the SM as a fundamental renormalizable theory, such fine tunings are of no concern. It is just the usual renormalization program. On the other hand, if one tries to build a viable theory with two scalar sectors at vastly different scales, like in GUT, then the hierarchy problem is back because radiative corrections tend to mix the scalar sectors together. Ensuring that the SM Higgs boson vacuum expectation value stays at around $10^2$ GeV instead of moving up to the GUT (or even Planck) scale requires totally unacceptable fine-tunings of the scalar potential. One solution to the hierarchy problem is for NP to arise close to the electroweak scale, with the hope that its dynamics somehow alleviates the fine tunings required in the scalar sector. An example of such a theory is supersymmetry [10].

- **Flavor replication:** The number of families is not dictated by any dynamical or symmetry principle in the SM. It is not constrained by the gauge or accidental symmetries, by renormalizability, or by any quantum field theory requirements. Even gauge anomalies cancel out separately for each family. All this remains true in most theories beyond the SM, leaving us without even the start of a clue as to why there should be three families. This is one of the most mysterious features observed in Nature. At the same time, because the Yukawa couplings are then arbitrary three-by-three matrices, they bring in around two-third of the SM free parameters. Furthermore, the fermion masses and mixings themselves are most puzzling: they clearly exhibit regular patterns. So, it is natural to expect that some high-scale dynamics is ultimately responsible for generating all these flavor structures. Such a mechanism should also explain why the low-scale NP required to solve the hierarchy problem (if it exists) has not been already discovered in flavor experiments. Hopefully, once this dynamics is identified, it will explain, or at least hint at the reason why the light fermions organize themselves into three generations.

- **Strong $\mathcal{CP}$ violation:** The current constraint on the neutron EDM require the effective $\theta$ term of QCD to be close to zero [11,12]. This is only possible provided the electroweak phase, coming from the Yukawa couplings, precisely cancel with the theta angle coming from the specific vacuum in which QCD happens to be realized. Solving this puzzle in general requires extending the SM particle content to renders the $\theta$ term dynamical, as for example introducing the so-called axions [13, 14].

- **Accidental symmetries:** The SM matter content happens to forbid gauge invariant and renormalizable couplings violating lepton ($\mathcal{L}$) or baryon ($\mathcal{B}$) numbers. Historically, global symmetries have proven very unreliable: either they end up broken explicitly, or need to be gauged. In the case of $\mathcal{B}$ and $\mathcal{L}$, the SM was proven to violate $U(1)_{\mathcal{B}+\mathcal{L}}$ through tiny non-perturbative effects [15]. Further, probably the most natural way to account for neutrino masses is the seesaw mechanism [1], in which a tiny Majorana mass [16] breaking $\mathcal{L}$ by two units is generated for left-handed neutrinos. So, there is really no reason to expect that $\mathcal{B}$ and $\mathcal{L}$ should be exactly realized in Nature. Yet, $\mathcal{B}$ and $\mathcal{L}$ violating effects have never been seen, with the bounds on proton decay [17] naively pushing the energy scale of such NP at or above the GUT scale.

In view of all these points, it is clear that some dynamics should complement (or entirely replace) that of the SM at some energy scale. Yet, we have no firm clue as to its main characteristics, or even its fundamental nature as a quantum field theory. All we can infer from our current theoretical understanding is that this new dynamics should kick in not too far from the TeV scale. From a pragmatic perspective, this would also be welcome since the LHC could then access it directly.

## Flavor perspectives on New Physics

Besides the LHC, low-energy experiments continue to play a central role even though their typical energy scale is very far from the TeV. There are two reasons for this counter-intuitive situation. Firstly, light leptons, baryons, or mesons can be copiously produced (in the case of electrons and protons, they are even readily available). Thanks to this, the luminosities achieved by these experiments are very high and the tiny virtual effects induced by the new heavy particles are a priori accessible. Secondly, some low-energy observables are extremely constrained in the SM by the peculiar hierarchies and accidental symmetries of its flavor sector. We can distinguish three broad classes: (1) Quark transitions through flavor-changing neutral currents and $\mathcal{CP}$-violation, which are correlated and suppressed in the SM. (2) Lepton flavor transitions, like $\mu \to e\gamma$ which are so small in the SM that they can be considered as forbidden. Similarly, the $\mathcal{CP}$-violating electric dipole moments (EDM) of the quarks and leptons are negligible in the SM. (3) Baryon and lepton number violating processes, like proton decay, neutron-antineutron oscillations, or neutrinoless double beta decays, which are strictly forbidden in the SM. By contrast, most NP models are much less restrictive. Actually, a generic NP model with rather light new particles, within reach of the LHC, typically violates the experimental constraints or bounds for all three classes of observables. This paradoxical situation is referred to as **the flavor puzzle** [18], and serves as the main motivation to introduce the **Minimal Flavor Violation** (MFV) hypothesis in Refs. [19, 20]. This is the topic of this report.

In the next Chapter, the flavor sector of the SM is detailed, adopting from the start the symmetry language on which MFV rests. In Chapter 2, the MFV hypothesis is introduced first in a model independent context, and then applied to the minimal supersymmetric standard model. As we will see, MFV first permits to characterize and quantify precisely the flavor puzzles by relying on an approximate symmetry principle. It then evolves into a pragmatic hypothesis which has to hold, at least approximately, on most TeV-scale NP model if they have to satisfy flavor constraints. In the final chapter, we will see that once taken seriously, MFV can also have far-reaching consequences, well beyond its planned domain of applicability. In particular, it could signal the demise of the conservation of baryon number. If true, a significant reappraisal of our NP search strategies at the LHC should be undertaken.

## Ph.D. student supervision in the MFV context

Before delving into the presentation, a word on the original contributions to this subject. The first chapter has evolved from lectures on the flavor puzzles given at Louvain University in 2008. The second chapter, where MFV is introduced, relies on results obtained in collaboration with Emanuel Nikolidakis and Lorenzo Mercolli, both Ph.D. students at Bern University at the time of my postdoc there. Specifically, the properties and resummation of MFV series, the classification of $\mathcal{CP}$-violating phases, and the presence of infrared fixed-point in the renormalization group evolution of MFV, where obtained in Refs. [21–23] and were incorporated in their theses defended in 2008 and 2010, respectively. These results have far reaching consequences, and are instrumental in developing our interpretation of MFV itself (for which we follow also Ref. [24]).

The idea of applying MFV to baryon number violating couplings, as presented in chapter 3, was first proposed in a paper with Emanuel Nikolidakis [25] (see also Ref. [26]). This implementation was then clarified in Ref. [27], and further improved recently in Ref. [28], written in collaboration with Jérémy Bernon, Ph.D. student in Grenoble. Besides, the phenomenological implications for the LHC, both model-independently [29] or within the MSSM [30], were studied together with Gauthier Durieux, Ph.D. student in Louvain. These analyses constituted the bulk of his thesis, defended in 2014. Currently, these studies are carried on with Simon Berlendis, Ph.D. student in the ATLAS group of the LPSC.

These works have been presented numerous times, and led to several proceedings authored by the students [31]. Carrier-wise, Emanuel Nikolidakis decided against starting postdoctoral studies, Lorenzo Mercolli got a Swiss National Fund grant to join the astrophysics group at Princeton University, while Gauthier Durieux got a grant from the Belgian American Education Foundation to go to Cornell University, and will move to DESY this Autumn to start his second postdoc.

# Chapter 1

# The Standard Model flavor sector

Two features of the SM are particularly mysterious. One is the Higgs mechanism: the spontaneous breaking of part of the SM gauge symmetry providing the necessary mass terms to the weak gauge bosons. This breaking is set up by the presence of a scalar field with coincidentally adequate values of its Lagrangian free parameters. Though this is a perfectly viable description, it is certainly not compelling and, as discussed in the introduction, cannot survive untouched to the presence of NP, even at a much higher scale. One thus remains with a sense that the story of the origin of masses is yet to be told. Unfortunately, initial clues from the LHC are not very helpful in driving us towards a deeper understanding, since all the measurements tend to confirm the SM picture.

The second particularly puzzling feature of the SM is its flavor sector, that is, its fermion content and the related parameters. As we will discuss in details in this report, the list of questions and puzzles is impressive:

1. Why are there three families of quark and leptons?
2. Why have the SM fermions just the right quantum numbers to cancel the gauge anomalies?
3. Why are the fermion masses so hierarchical, and the quark interfamily transitions so suppressed?
4. Why is the $\mathcal{CP}$ symmetry violated in the weak interactions, and why is it apparently driven by a unique parameter?
5. Why are individual charged lepton flavors conserved to an excellent approximation?
6. Why are neutrino (quasi)massless? Are they Dirac or Majorana particles?
7. What drives the strong $\mathcal{CP}$-violation towards zero?
8. Why are lepton and baryon numbers so nearly conserved? Are they truly violated only by tiny non perturbative weak interactions effects?

Most of these puzzles are experimentally driven. Theory does not tell us anything about the number of families, the quark and lepton masses and mixing patterns, or the magnitude of $\mathcal{CP}$ violation. Those are free parameters which have been measured or constrained quite precisely. Also, the unsuccessful searches for $\mu \to e\gamma$ (lepton flavor violation?), neutrinoless double beta decay (Majorana neutrinos?), neutron electric dipole moment (strong $\mathcal{CP}$ violation?), or proton decay ($\mathcal{B}$ and $\mathcal{L}$ violation?) have reached such incredible sensitivities that they cannot be purely coincidental and left unexplained.

The presence of three families stands out among these questions. On the one hand, finding a mechanism responsible for the fermion replication would certainly be a discovery of great significance, with deep implications on our understanding of the constituents of the Universe. On the other hand, this is the only puzzle of the list for which the absence of a true explanation would not be too shocking. One could easily accept that Nature just happens to be that way. To some extent, one may adopt a similar attitude towards $\mathcal{CP}$ violation. It occurs in the SM because with three families of quarks, a complex parameter remains in the Lagrangian once all fermion phase redefinitions have been performed. At the same time, $\mathcal{CP}$ is intimately related to the properties of the space-time fabric of our Universe, so accepting its violation on the basis of a mere redefinition leftover may not be entirely convincing.

All the other puzzles clearly beg for dynamical explanations, and thus ask for extending the SM in one way or another. For example, the second question may resolved itself if the SM gauge symmetry originates as a spontaneously broken anomaly-free semi-simple group like in the $SO(10)$ GUT models, the neutrino masses are automatically small if the seesaw mechanism [1] is brought in along with its triplet of very heavy right-handed neutrinos, while the strong $\mathcal{CP}$ puzzle may find its demise in the axion dynamics [13,14]. But once accepted that the SM needs to be extended, the other questions which were coincidentally solved in the SM come back haunting us. For instance, the very peculiar quark and lepton mixing patterns, the uniqueness of the $\mathcal{CP}$ violation source, and the $\mathcal{B}$ and $\mathcal{L}$ accidental symmetries are usually lost, in direct conflict with experimental results which confirm the SM picture to an impressive level of precision.

Intriguingly, most of these flavored issues find their origins in the couplings of fermions with the Higgs boson. The Higgs mechanism is responsible for fermion masses, and thereby their mixing, as well as for making the strong $\mathcal{CP}$ puzzle so puzzling. Besides, the discrete symmetries $\mathcal{CP}$, $\mathcal{B}$ and $\mathcal{L}$ happen to all be violated by the weak interaction, which is precisely the one gauge symmetry undergoing the spontaneous breaking. The two most mysterious features of the SM thus seem to share a deep connection.

In the present chapter, the peculiarities of the SM flavor sector are presented, leaving the discussion of $\mathcal{B}$ and $\mathcal{L}$ to Chapter 3. Most of the material covered here is fairly basic and not really original. However, we will adopt a particular language, that of the flavor symmetry and its breaking. Such a systematic use of this symmetry has to our knowledge never been carried out. Not only will it shed new lights on old problems, it will also be central in the discussion of constraints on NP and of MFV undertaken in Chapter 2. Further, from a pedagogical point of view, it is a very convenient organizing principle, and once its associated techniques are mastered, a very effective tool to develop one's intuition.

## 1.1 Flavor couplings and free parameters

The SM is defined upon a gauge symmetry under the group $G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, corresponding to the strong, weak, and hypercharge interactions. The couplings with matter fields follow from the minimal substitution principle

$$\mathcal{L}_{\text{fermion}} = \sum_{\psi,I} \bar{\psi}^I (i\not{D}_\psi)\psi^I, \quad \psi = Q, U, D, L, E, \quad I = 1,2,3\ , \tag{1.1}$$

with the covariant derivative

$$D^\mu_\psi = \partial^\mu - ig_s T^a_\psi G^\mu_a - ig\mathbf{T}_\psi \cdot \mathbf{W}^\mu - ig' \frac{Y_\psi}{2} B^\mu\ , \tag{1.2}$$

where $(T^a_\psi, \mathbf{T}_\psi, Y_\psi/2)$ is the group generator for the $G_{SM}$ representation carried by the field $\psi$, as collected in Table 1.1. These covariant derivatives do not depend on the flavor index $I$, i.e., the gauge interactions are independent of the flavor of the fermion fields. Therefore, $\mathcal{L}_{\text{fermion}}$ is invariant under

| Matter | $\mathcal{B}$ | $\mathcal{L}$ | $G_F$ | $G_{SM}:$ | $T^a_\psi$ | $\mathbf{T}_\psi$ | $Y_\psi/2$ |
|---|---|---|---|---|---|---|---|
| $Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$ | 1/3 | 0 | $(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$ | $(\mathbf{3},\mathbf{2})_{+1/3}:$ | $+\lambda^a/2$ | $\boldsymbol{\sigma}/2$ | $+1/6$ |
| $U = u_R^\dagger$ | $-1/3$ | 0 | $(\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1})$ | $(\mathbf{3},\mathbf{1})_{-4/3}:$ | $-\lambda^a/2$ | $\mathbf{0}$ | $-2/3$ |
| $D = d_R^\dagger$ | $-1/3$ | 0 | $(\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{1})$ | $(\mathbf{3},\mathbf{1})_{+2/3}:$ | $-\lambda^a/2$ | $\mathbf{0}$ | $+1/3$ |
| $L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ | 0 | 1 | $(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{1})$ | $(\mathbf{1},\mathbf{2})_{-1}:$ | 0 | $\boldsymbol{\sigma}/2$ | $-1/2$ |
| $E = e_R^\dagger$ | 0 | $-1$ | $(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{3})$ | $(\mathbf{1},\mathbf{1})_{+2}:$ | 0 | $\mathbf{0}$ | $+1$ |

Table 1.1: The gauge quantum numbers of the SM fermions and their associated $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ generators ($\lambda^a$ denote the eight Gell-mann matrices and $\boldsymbol{\sigma}$ the three Pauli matrices). In the second and third columns are indicated the baryon and lepton numbers, respectively. Note that the right-handed fermions are defined by their left-handed conjugate Weyl spinors. Throughout this work, the appropriate translation to the Dirac notation is always understood, with e.g. $U$ standing either for $u_R^\dagger$, $\bar{u}_R$, or $u_R^C$, which all transform under the same $G_{SM} \otimes G_F$ representation.

an arbitrary renaming of the fermion flavors; the distinction between e.g. $u, c$, and $t$ is just a matter of convention. Formally, this is expressed as an invariance under independent unitary transformations of each of the five matter fields:

$$\psi^I \to (g_\psi)^{IJ}\psi^J : \mathcal{L}_{\text{fermion}} \to \sum_{\psi,I,J,K} \bar{\psi}^K (g_\psi^\dagger)^{KI}(i\not{D}_\psi)(g_\psi)^{IJ}\psi^J = \mathcal{L}_{\text{fermion}} \ , \tag{1.3}$$

where $g_\psi$ are $U(3)$ matrices, such that $(g_\psi^\dagger)^{IK}(g_\psi)^{KJ} = \delta^{IJ}$. The gauge sector of the SM is thus invariant under the large global symmetry group [32]

$$G_F = U(3)^5 = U(3)_Q \otimes U(3)_U \otimes U(3)_D \otimes U(3)_L \otimes U(3)_E \ , \tag{1.4}$$

which is called the **flavor symmetry**. The action of this group is defined such that left-handed fermions transform as $\mathbf{3}$ under their respective $U(3)$, i.e., $X \to g_X X$ for $X = Q, L$ and $X \to X g_X$ for $X = U, D, E$, see Table 1.1.

This symmetry is not exact in the SM though. It is explicitly broken by the couplings of fermions with the Higgs field $H \sim (\mathbf{1},\mathbf{2})_{+1}$,

$$\mathcal{L}_{\text{Yukawa}} = -U^I \mathbf{Y}_u^{IJ} Q^J H^{\dagger C} - D^I \mathbf{Y}_d^{IJ} Q^J H^\dagger - E^I \mathbf{Y}_e^{IJ} L^J H^\dagger + h.c. \ . \tag{1.5}$$

Clearly, these fermion-fermion-scalar couplings, called **Yukawa couplings**, break $G_F$ since they mix different species of fermions. This is the source of most of the free parameters of the SM. Indeed, because there are three families, $\mathbf{Y}_{u,d,e}$ are $3 \times 3$ complex matrices in flavor space hence this sector introduces $3 \times 18 = 54$ real parameters. For comparison, the rest of the SM introduces only 6 free parameters: the three gauge couplings, the Higgs boson quadratic and quartic term, and the strong $\theta$ term. Fortunately, not all of these 54 parameters are physical. After all field redefinitions have been performed, one remains with a total of 13 free parameters for $\mathcal{L}_{\text{Yukawa}}$. This is better, but still represents about two-third of the SM free parameters.

Let us analyze these redefinitions and the corresponding counting of the free parameters from the point of view of the flavor symmetry [33]. First, it must be remarked that $G_F$ is not entirely broken by the Yukawa couplings, which leave out the **accidental symmetries** of the SM:

$$U(3)^5 \to U(1)_{\mathcal{B}} \otimes U(1)_{\mathcal{L}_e} \otimes U(1)_{\mathcal{L}_\mu} \otimes U(1)_{\mathcal{L}_\tau} \ , \tag{1.6}$$

where $\mathcal{B}$ is the **baryon number** and $\mathcal{L}_{e,\mu,\tau}$ are the three **lepton flavor numbers**. Specifically, the invariance under $U(1)_{\mathcal{B}}$, whose charges are $\mathcal{B}(Q^I, U^{\dagger I}, D^{\dagger I}) = +1/3$ and $\mathcal{B}(Q^{I\dagger}, U^I, D^I) = -1/3$, follows from the fact that all SM couplings involve as many quarks as antiquarks. For the lepton flavor numbers, which are defined as $\mathcal{L}_{e^I}(L^J, E^{J\dagger}) = \delta^{IJ}$ and $\mathcal{L}_{e^I}(L^{J\dagger}, E^J) = -\delta^{IJ}$, the invariance is less obvious looking at Eq. (1.5) but will become clear later on. Note that the **lepton number** is defined as the sum $\mathcal{L} \equiv \mathcal{L}_e + \mathcal{L}_\mu + \mathcal{L}_\tau$, and is also trivially conserved in the SM. Given this pattern of explicit symmetry breaking, the counting is very simple. There are $3 \times 2 \times 9 = 54$ free real parameters from $\mathbf{Y}_{u,d,e}$, while a $G_F$ transformation involves 5 unitary matrices, each with nine real parameters. Removing the four unbroken $U(1)$s, which cannot absorb anything since they remain exact, the flavor symmetry permits to remove $5 \times 9 - 4 = 41$ real parameters, leaving 13 physical real parameters.

Of course, the flavor symmetry does not tell us what are those 13 parameters. To identify them, let us go back to the Yukawa couplings. After the electroweak Spontaneous Symmetry Breaking (SSB), the Higgs field acquires a real vacuum expectation value, $\langle 0|H^0|0\rangle = v$, and can be written in the unitary gauge as $H = (0, v+h)^T$ with $h$ the physical Higgs boson. Upon this shift, the Yukawa couplings give rise to the fermion mass terms

$$\mathcal{L}_{\text{Yukawa}} = -v\left(\bar{u}_R^I \mathbf{Y}_u^{IJ} u_L^J + \bar{d}_R^I \mathbf{Y}_d^{IJ} d_L^J + \bar{e}_R^I \mathbf{Y}_e^{IJ} e_L^J\right)\left(1 + \frac{h}{v}\right) + h.c. \,. \tag{1.7}$$

A priori, none of these couplings is diagonal in flavor space. The fermion mass eigenstates are obtained through biunitary rotations,

$$v V_R^{u,d,e} \mathbf{Y}_{u,d,e} V_L^{u,d,e} = \mathbf{m}_{u,d,e} \;, \tag{1.8}$$

where the mass matrices $\mathbf{m}_{u,d,e}$ are diagonal. Such a procedure, called the **singular value decomposition** (SVD), is always possible. For a generic complex matrix $\mathbf{M}$, we can find two unitary matrices such that $V\mathbf{M}U^\dagger = \mathbf{D}$ where $\mathbf{D}$ is a diagonal matrix with real entries greater or equal to zero, the singular values. To reach this form, it suffices to consider $\mathbf{M}^\dagger\mathbf{M}$ and $\mathbf{M}\mathbf{M}^\dagger$, which are hermitian hence diagonalizable with the unitary matrices $U$ and $V$, respectively.

The unitary transformations required for the singular value decomposition in Eq. (1.8) belong to the $G_F$ symmetry, but for the left-handed quark fields. The gauge symmetry forces us to rotate the whole quark doublet $Q^I$, i.e., to rotate $u_L^I$ and $d_L^I$ by the same unitary matrix. Because $V_L^u \neq V_L^d$ in general, the symmetry of the gauge sector is not sufficient to bring both $\mathbf{Y}_u$ and $\mathbf{Y}_d$ in diagonal form. Said differently, the fermion gauge and mass eigenstates are irremediably different. Let us make a choice, and rotate $Q$ by $V_L^{u\dagger}$. Then, the best we can do without breaking $G_F$ is to put all quarks in their mass eigenstates except for the left-handed down quarks $d_L$, $s_L$, and $b_L$. In other words, using the $U(3)^5$ symmetry of the gauge interactions, we can reach the (gauge-)basis

$$\mathcal{L}_{\text{Yukawa}} = -\left(\bar{u}_R \mathbf{m}_u u_L + \bar{d}_R \mathbf{m}_d V_{CKM}^\dagger d_L + \bar{e}_R \mathbf{m}_e e_L\right)\left(1 + \frac{h}{v}\right) + h.c. \,, \tag{1.9}$$

where the rotation still needed to reach the mass eigenstates is conventionally defined as

$$V_L^{u\dagger} V_L^d \equiv V_{CKM} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \,, \tag{1.10}$$

and is called the **Cabibbo-Kobayashi-Maskawa matrix**, or CKM in short [34].

Experimentally, the breaking of $G_F$ induced by $\mathbf{Y}_{u,d,e}$ turns out to be far from generic or natural. First, the fermion masses show a very strong hierarchy with for example $m_u/m_t \sim 10^{-5}$, and second, the CKM matrix deviates only moderately from a trivial unit matrix. Before quoting the numerical values for the CKM parameters, it is necessary to precisely identify them, which requires adopting some conventions. The CKM matrix starts as a three-by-three unitary matrix, so with 9 free real parameters. From the abstract counting made above, only four of them should be physical since 9 out of the 13 free parameters correspond to the fermion masses. In practice, five phases can be absorbed in the quark wavefunctions. This is embedded in the singular value decomposition, where $V$ and $U$ are defined up to a diagonal matrix of phases[1]. If we define $D^{u,d} \equiv \text{diag}(e^{i\alpha_1^{u,d}}, e^{i\alpha_2^{u,d}}, e^{i\alpha_3^{u,d}})$, then

$$vD^{u\dagger}V_R^u\mathbf{Y}_uV_L^uD^u = \mathbf{m}_u \ , \ vD^{d\dagger}V_R^d\mathbf{Y}_dV_L^dD^d = \mathbf{m}_d \ , \tag{1.11}$$

leaves the singular values (i.e., the masses) untouched but changes the CKM matrix as $V_{CKM} \to V'_{CKM} = D^{u\dagger}V_{CKM}D^d$. Through this innocuous redefinition of $V_{L,R}^{u,d}$, five of the $V_{CKM}$ phases can be removed since the transformation with all the $\alpha_i^{u,d}$ equal leave $V_{CKM}$ invariant. Adopting the standard convention for the five phase redefinitions, the CKM matrix can be expressed in terms of Wolfenstein parameters [35] as

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3\,(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3\,(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) \ , \tag{1.12}$$

with the current best-fit [36] values $\lambda = 0.22548^{+0.00068}_{-0.00034}$, $A = 0.810^{+0.018}_{-0.024}$, $\bar{\rho} \equiv \rho(1-\lambda^2/2) = 0.145^{+0.013}_{-0.007}$, and $\bar{\eta} \equiv \eta(1-\lambda^2/2) = 0.343^{+0.011}_{-0.012}$. Note in particular how the unique source for all the weak $\mathcal{CP}$-violation, the imaginary parts $i\eta$, occurs in suppressed $\mathcal{O}(\lambda^3)$ entries.

## 1.2 Flavor-changing charged and neutral currents

Fermion mass eigenstates are not aligned with gauge eigenstates, so the flavor of a fermion is not a good quantum number. However, the situation in the SM is quite peculiar in several respects. First, to get the mass eigenstates starting from the gauge basis of Eq. (1.9), we need to break the $SU(2)_L$ symmetry and rotate $d_L \to V_{CKM}d_L$ while keeping $u_L$ fixed. This rotation directly affects the **quark charged currents**[2], but leaves the lepton current untouched:

$$\mathcal{L}_{\text{CC}} = \frac{g}{\sqrt{2}}\sum_I W_\mu^+\left(\bar{\nu}_L^I\gamma^\mu e_L^I + \bar{u}_L^I\gamma^\mu V_{CKM}^{IJ}d_L^J\right) + h.c. \ . \tag{1.13}$$

The lepton couplings are immediately diagonal in flavor space because $\mathbf{Y}_e$ can be diagonalized without breaking $G_F$. This is the reason why the three lepton flavor numbers are separately conserved, corresponding to the residual $U(1)_{\mathcal{L}_e} \otimes U(1)_{\mathcal{L}_\mu} \otimes U(1)_{\mathcal{L}_\tau}$ accidental symmetry. **Lepton Flavor Violating** (LFV) processes like $\mu \to e\gamma$, $eee$, $e\gamma\gamma$ are strictly forbidden in the SM. A second feature of the SM is to keep all the **neutral currents** diagonal in flavor space at the Lagrangian level. This is trivial for the neutral Higgs boson coupling, see Eq. (1.9). For the $Z$ boson and photon couplings to $d_L$ quarks,

$$\mathcal{L}_{\text{NC}} \ni \frac{g}{2\cos\theta_W}Z_\mu\bar{d}_L^I\gamma^\mu\left(2T_3 - 2\sin^2\theta_W Q\right)d_L^I + eA_\mu\bar{d}_L^I\gamma^\mu Q d_L^I \ , \tag{1.14}$$

in the gauge basis, the invariance when rotating to the mass basis $d_L \to V_{CKM}d_L$ follows from the unitarity of the CKM matrix.

At leading order, charged currents are thus our only window to the CKM matrix, and thereby the only pathway through which $\mathcal{CP}$ violation can find its way into observables. Since $V_{CKM}$ is close to

[1] If some singular values are equal, there is a larger ambiguity corresponding to any rotation in the degenerate subspace.

[2] If we had chosen to put the CKM matrix with $\mathbf{Y}_u$, i.e., to work with $d_L$ in their mass eigenstates instead of $u_L$, this same coupling would then be obtained through $u_L \to V_{CKM}^\dagger u_L$, which implies $\bar{u}_L \to \bar{u}_L V_{CKM}$.

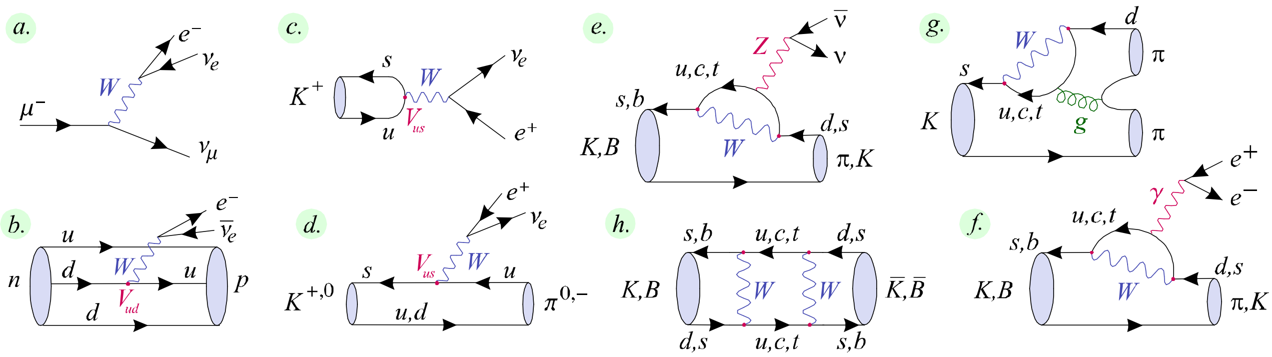


Figure 1.1: $(a-d)$ The charged-current interactions. In the quark sector, it is tuned by the CKM matrix and can induced flavor-changing transitions. $(e-h)$ The Flavor-changing neutral currents arising at the loop level in the SM. Depicted are the $Z$, $\gamma$, gluon penguin $(e,f,g)$ and the $W$ box $(h)$.

the unit matrix, these flavor mixing are suppressed. Experimentally, the charged currents are probed at low energy through flavor changing processes (see Fig. 1.1), like the leptonic decays $K, D, B \to \ell\nu$, semileptonic processes like $K, D, B \to \pi\ell\nu$, etc. From these modes, upon knowledge of the hadronic matrix elements, e.g. $\langle 0|\bar{u}_L\gamma_\mu s_L|K^+\rangle$, the absolute values of most CKM matrix elements can be extracted.

Beyond leading order, the flavor-mixings present in the charged currents spill to the neutral currents, and therefore tune the **Flavor-Changing Neutral Currents** (FCNC). Being in addition suppressed, these FCNC offer unique and non-trivial tests of the SM (for an introduction and review, see e.g. Ref. [37]). Specifically, consider the diagrams shown in Fig. 1.1, called **penguins** and **boxes**, and concentrate on the $Z$ penguin. Thanks to the $W$ boson, it is now possible to go from one generation of down (or up) quark to another. Taking the $s \to d$ transition for definiteness, and summing over the intermediate $u, c, t$ quarks, the total amplitude is

$$\mathcal{M}(\bar{s}d \to Z) = V_{ud}V_{us}^* f(m_u/M_W) + V_{cd}V_{cs}^* f(m_c/M_W) + V_{td}V_{ts}^* f(m_t/M_W)\ . \tag{1.15}$$

The **Inami-Lim function** $f(m_q/M_W)$ keeps track of the dependence of the loop integral on the virtual particle masses [38]. These dependences are crucial: if $f(m_q/M_W)$ is constant, as would occur if the quark masses were equal, it could be factored out and $\mathcal{M}(\bar{s}d \to Z) = 0$ since the CKM matrix is unitary, $V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0$. This is the **Glashow-Iliopoulos-Maiani (GIM) mechanism** [39]. It is only through a delicate breaking of the degeneracy among quark masses that FCNC transitions are possible.

The most interesting penguin processes are those coupled to the $Z$ boson. Indeed, compared to the photon penguin, for which the QED Ward identity requires

$$\mathcal{M}\left(\bar{s}d \to \gamma^*(q)\right) \sim G_F \times \frac{e}{4\pi^2} \times \bar{s}_L\gamma_\mu d_L \times \left(q^\mu q^\nu - q^2 g^{\mu\nu}\right) \times A_\nu \times \sum_q V_{qs}^* V_{qd}\, F_\gamma(m_q^2/M_W^2)\ , \tag{1.16}$$

the corresponding $Z$-boson Ward identity is broken. Therefore, it is possible to trade the $q^\mu q^\nu - q^2 g^{\mu\nu}$ projector for the $SU(2)_L$-breaking parameter $v$, with a net enhancement by $v^2/q^2 \gg 1$ since the exchanged momentum $q \sim m_K, m_B$ is small. The simplest and largest term originates from a double quark mass insertions (each breaking $SU(2)_L$)

$$\mathcal{M}\left(\bar{s}d \to Z(q)\right) \sim G_F \times \frac{e}{4\pi^2 \sin\theta_W} \times \bar{s}_L\gamma_\mu d_L \times Z^\mu \times \sum_q m_q^2\, V_{qs}^* V_{qd}\, F_Z(m_q^2/M_W^2)\ . \tag{1.17}$$

In this case, one speaks of quadratic violation of the GIM mechanism. The theoretically cleanest processes in which the $Z$ penguin dominates are the $B_{d,s} \to \mu^+\mu^-$ and $K \to \pi\nu\bar{\nu}$ decays, though many other less accurate modes like $B \to \pi K$, $K_L \to \mu^+\mu^-$, etc, are also used to provide some constraints.

Though the virtual photon penguin is not competitive compared to the $Z$ penguin, the real photon penguin has a unique signature easing its experimental observation. The amplitude is suppressed by one external quark chirality flip (i.e., left-right mixing)

$$\mathcal{M}\left(\bar{s}d \to \gamma\left(q\right)\right) \sim G_F \times \frac{e}{4\pi^2} \times m_s \times \bar{s}_R \sigma_{\mu\nu} d_L \times F^{\mu\nu} \times \sum_q V_{qs}^* V_{qd}\, F_\gamma'(m_q^2/M_W^2) \,. \tag{1.18}$$

This penguin is best probed with $B$ decays since $m_b \gg m_s$. Currently, the inclusive $b \to s\gamma$ process is measured in reasonable agreement with the SM prediction. The last kind of penguins are the gluonic ones, which have amplitudes similar to those for the real and virtual photons. The techniques required to deal with these purely strong interacting penguins are more involved and will not be discussed here.

The other class of FCNC are the boxes, directly relevant for $B_d - \bar{B}_d$, $B_s - \bar{B}_s$, and $K - \bar{K}$ particle-antiparticle mixings (see Fig. 1.1$c$). The corresponding effective interactions also show a quadratic violation of the GIM mechanism, hence retaining only the top quark contribution:

$$\mathcal{M}\left(\bar{s}d \to \bar{d}s\right) = \frac{G_F^2 m_t^2}{4\pi^2} \times (V_{ts}V_{td}^*)^2 \times B_{WW}(m_t^2/M_W^2) \times (\bar{s}_L\gamma^\mu d_L)\,(\bar{s}_L\gamma_\mu d_L) \;. \tag{1.19}$$

Contrary to the $Z$-penguin, this factor of $m_t^2 \sim v^2$ should however not be thought of as an $SU(2)_L$ breaking in $v^2$ since $G_F^2 m_t^2 \sim G_F \times m_t^2/v^2$. From this, including QCD corrections as well as lattice estimates for the hadronic matrix element, the top-quark contribution[3] is predicted at about $|\varepsilon_K| \approx 3 \cdot 10^{-3}$, which agrees reasonably well with the measurement $|\varepsilon_K|^{\exp} = (2.232 \pm 0.007) \times 10^{-3}$ [17].

## 1.3 How to exploit the flavor symmetry in a clever way?

Additional insight into the structures of the FCNC in the SM can be gained using the symmetry $G_F$. Since it is broken only by the Yukawa couplings in the SM, it is formally restored if these Yukawa couplings are given definite transformation rules under $G_F$, i.e., are promoted to **spurions**. Looking at Eq. (1.5) and with the convention that $Q, U, D, L, E$ transform as $\mathbf{3}$ under their respective $SU(3)$, the whole SM Lagrangian becomes invariant under $G_F$ if

$$\mathbf{Y}_u \sim (\bar{\mathbf{3}}, \bar{\mathbf{3}}, \mathbf{1}, \mathbf{1}, \mathbf{1})_{G_F} : \mathbf{Y}_u \overset{G_F}{\to} g_U^\dagger \mathbf{Y}_u g_Q^\dagger \,, \tag{1.20a}$$

$$\mathbf{Y}_d \sim (\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}}, \mathbf{1}, \mathbf{1})_{G_F} : \mathbf{Y}_d \overset{G_F}{\to} g_D^\dagger \mathbf{Y}_d g_Q^\dagger \,, \tag{1.20b}$$

$$\mathbf{Y}_e \sim (\mathbf{1}, \mathbf{1}, \mathbf{1}, \bar{\mathbf{3}}, \bar{\mathbf{3}})_{G_F} : \mathbf{Y}_e \overset{G_F}{\to} g_E^\dagger \mathbf{Y}_e g_L^\dagger \,, \tag{1.20c}$$

where $g_X \in U(3)_X$. This is a purely formal manipulation, but it will prove extremely fruitful. Indeed, as soon as the SM Lagrangian becomes invariant under $G_F$, even if this is purely artificial, the amplitude for any possible process must also be expressible as manifestly $G_F$-invariant. Crucially, this invariance may require inserting Yukawa spurions in a very specific way in the amplitude. Its flavor structure can thus be established quite precisely without embarking into any computation. This even translates into quantitative predictions once the spurions are frozen back to their physical values, for example to

$$v\mathbf{Y}_u \overset{frozen}{\to} \mathbf{m}_u V_{CKM},\;\; v\mathbf{Y}_d \overset{frozen}{\to} \mathbf{m}_d,\;\; v\mathbf{Y}_e \overset{frozen}{\to} \mathbf{m}_e \,, \tag{1.21}$$

[3] The parameter $\varepsilon_K$ describes the mismatch of the mass eigenstates, $K_L$ and $K_S$, with respect to the pure CP-eigenstates, $(K^0 \pm \bar{K}^0)/2$. It can be extracted from $K_{L,S} \to \pi\pi$ measurements [17].

in the gauge-basis in which all but the $u_L$ quarks are mass eigenstates (that with all but $d_L$ quarks would move the $V_{CKM}$ factor into $\mathbf{Y}_d$). In this way, some processes are immediately predicted to be very suppressed compared to others as a result of the very peculiar numerical hierarchies present in the Yukawa couplings. Also, it is immediate to see that no leptonic FCNC are allowed since after inserting $\mathbf{Y}_e$ in an amplitude with external lepton fields, it gets frozen to its diagonal background $\mathbf{m}_e/v$. A non-trivial neutrino sector is compulsory to get some LFV effects.

### 1.3.1 How to predict the scaling of flavor-changing currents

This set of tricks is central to the later discussion of MFV, so let us illustrate in some details how to apply them for the FCNC processes discussed in the previous section. We start with the virtual photon penguin. To be able to use the $G_F$ symmetry of the gauge sector, it better not be spontaneously broken yet. Also, all renormalizable dimension-four couplings are already part of the SM Lagrangian so the effective interactions corresponding to the virtual photon penguins must be of higher dimension. The simplest possible such interactions are (remember $D\gamma_\mu Q = \bar{d}_R\gamma_\mu q_L = 0$)

$$\mathcal{L}_{\text{eff}} = G_F a_1 \times (\bar{Q}\gamma_\nu Q)D_\mu F^{\mu\nu} + G_F a_2 \times (D\gamma_\nu \bar{D})D_\mu F^{\mu\nu} \;, \tag{1.22}$$

where $a_{1,2}$ are some numbers, a priori of $\mathcal{O}(1)$ [4]. Note that effective interactions involving derivatives acting on quark fields vanish upon using the Dirac equation since before SSB, all the quarks are massless. The $D_\mu F^{\mu\nu}$ part of these interactions correctly reproduces the $q^\mu q^\nu - q^2 g^{\mu\nu}$ projector in Eq. (1.16), but at this level $\mathcal{L}_{\text{eff}}$ is still diagonal in flavor space. To mix the flavors, we must insert some Yukawa spurions. Given the transformation rules in Eq. (1.20), this can be done as

$$\bar{Q}^I\gamma_\nu Q^I \to \bar{Q}^I(\mathbf{1}\oplus\mathbf{Y}_d^\dagger\mathbf{Y}_d\oplus\mathbf{Y}_u^\dagger\mathbf{Y}_u\oplus...)^{IJ}\gamma_\nu Q^J \;, \tag{1.23}$$

$$D^I\gamma_\nu\bar{D}^I \to D^I(\mathbf{1}\oplus\mathbf{Y}_d\mathbf{Y}_d^\dagger\oplus\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger\oplus...)^{IJ}\gamma_\nu\bar{D}^J \;, \tag{1.24}$$

where the $\oplus$'s serve as reminders that different $\mathcal{O}(1)$ numbers may appear as coefficients for each term of these expansions. Once the spurions have been appropriately introduced, they are frozen to their physical values in some gauge basis. When transitions between on-shell down-type quarks are considered, the values of Eq. (1.21) are appropriate, and the structure $\mathbf{Y}_u^\dagger\mathbf{Y}_u$ emerges as the one able to induce flavor transitions since it is not diagonal. Interestingly, it even correctly account for the GIM mechanism. If quark masses were equal, $\mathbf{m}_u = m\mathbf{1}$, then we would get $v^2\mathbf{Y}_u^\dagger\mathbf{Y}_u = m^2\mathbf{1}$ because of the unitarity of the CKM matrix. So, $\mathbf{Y}_u^\dagger\mathbf{Y}_u$ embodies a quadratic breaking of the GIM mechanism, induced by the large top mass $m_t$:

$$v^2\mathbf{Y}_u^\dagger\mathbf{Y}_u \approx m_t^2\begin{pmatrix} |V_{td}|^2 & V_{td}^*V_{ts} & V_{td}^*V_{tb} \\ V_{ts}^*V_{td} & |V_{ts}|^2 & V_{ts}^*V_{tb} \\ V_{tb}^*V_{td} & V_{tb}^*V_{ts} & |V_{tb}|^2 \end{pmatrix} \;. \tag{1.25}$$

Using this to estimate the strength of the effective interactions, we find

$$\bar{Q}^I(\mathbf{Y}_u^\dagger\mathbf{Y}_u)^{IJ}\gamma_\nu Q^J \to \frac{m_t^2}{v^2}V_{tI}^\dagger V_{tJ}\otimes(\bar{d}_L^I\gamma_\nu d_L^J) \;, \tag{1.26a}$$

$$D^I(\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger)^{IJ}\gamma_\nu\bar{D}^J \to \frac{m_{d^I}m_{d^J}}{v^2}\frac{m_t^2}{v^2}V_{tI}^\dagger V_{tJ}\otimes(\bar{d}_R^I\gamma_\nu d_R^J) \;. \tag{1.26b}$$

So, using only the flavor symmetry, we are able to correctly predict not only the CKM scaling of the FCNC transitions, but also the chirality flips. In the above case, the second operator involving right-handed quarks requires two such flips because the $W$ boson couples only to left-handed fermions. As a result, it is always very suppressed compared to the first one since $m_{d,s,b} \ll v$.

[4] In writing $D\gamma_\mu\bar{D}$, we introduced a slightly non-conventional notation $D \equiv \bar{d}_R$ and $\bar{D} \equiv d_R$.

Consider now the $Z$-boson penguin. We know from the previous section that operators like $(\bar{Q}\gamma_\nu Q)D_\mu Z^{\mu\nu}$ are suppressed compared to the $SU(2)_L$ breaking operators by $q^2/v^2$ with $q \sim m_K$ or $m_B$. Since here we only consider $SU(2)_L \otimes U(1)_Y$ gauge invariant operators, how does this $SU(2)_L$-breaking contribution arise? Necessarily, it must involve the external would-be Goldstone bosons, i.e. the Higgs fields:

$$\mathcal{L}_{\text{eff}} = G_F a_1 \times (\bar{Q}\mathbf{Y}_u^\dagger \mathbf{Y}_u \gamma_\mu Q) H^\dagger D^\mu H + G_F a_2 \times (\bar{Q}\mathbf{Y}_u^\dagger \mathbf{Y}_u \gamma_\mu \sigma^i Q)(H^\dagger \sigma^i D^\mu H) \; , \tag{1.27}$$

where we have already dropped $D\gamma_\mu \bar{D}$, suppressed by down-quark masses, and inserted the appropriate Yukawa spurions. After SSB, the first coupling has a $v^2$-enhanced piece

$$H^\dagger D^\mu H \overset{\text{SSB}}{\to} i\frac{1}{2}v^2 \left(gW_3^\mu - g' B^\mu\right) = iv^2 \frac{g}{2\cos\theta_W} Z^\mu \; , \tag{1.28}$$

and combined with Eq. (1.26), we recover precisely Eq. (1.17). There is no piece proportional to the photon field since $U(1)_{em}$ is unbroken. For the second coupling, only $\sigma^3$ contributes and leads to the same final contribution.

Finally, consider the magnetic photon penguin operators, which is the only other dimension-six operator constructible from $F^{\mu\nu}$

$$(D\mathbf{Y}_d \sigma_{\mu\nu} Q) H^\dagger F^{\mu\nu} \overset{SSB}{\to} (D\mathbf{m}_d \sigma_{\mu\nu} Q) F^{\mu\nu} \; . \tag{1.29}$$

After SSB, the helicity suppression factor $m_d$ is automatically generated. As before, to mix the flavors, we must insert some $\mathbf{Y}_u$ in a $G_F$-invariant way, e.g. as $(D\mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u \sigma_{\mu\nu} Q) H^\dagger F^{\mu\nu}$, and we recover the structure found in Eq. (1.18).

In addition to the penguin operators, there are all the possible four-fermion operators involving leptons and/or quarks, corresponding to the box diagrams. Let us just consider those relevant for $K - \bar{K}$ or $B - \bar{B}$ mixing:

$$\mathcal{L}_{\text{eff}} = G_F^2 b_1 \times (\bar{Q}\mathbf{Y}_u^\dagger \mathbf{Y}_u \gamma_\mu Q)(\bar{Q}\mathbf{Y}_u^\dagger \mathbf{Y}_u \gamma^\mu Q) + G_F^2 b_2 \times \varepsilon^{\alpha\beta} (\bar{Q}_\alpha \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger \bar{D})(D\mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u Q_\beta) + \ldots \; . \tag{1.30}$$

The second operator is suppressed because of the double $\mathbf{Y}_d$ insertion, while the first is proportional to $(V_{tI}^\dagger V_{tJ})^2 \times (\bar{d}_L^I \gamma_\nu d_L^J)^2$, as found in Eq. (1.19).

### 1.3.2 How to predict the EDM induced by the $\mathcal{CP}$-violating CKM phase

The flavor-symmetry formalism can also be used for flavor-conserving observables. For instance, consider the flavor-diagonal photon penguin whose general structure is (remember $2\sigma^{\mu\nu}\gamma_5 = i\varepsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta}$ and $\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta} \equiv 2\tilde{F}^{\mu\nu}$)

$$\begin{aligned}\mathcal{H}_{eff} &= c\,\bar{\psi}_L \sigma^{\mu\nu} \psi_R F_{\mu\nu} + c^*\,\bar{\psi}_R \sigma^{\mu\nu} \psi_L F_{\mu\nu} \\ &= (\operatorname{Re} c)\,\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu} + i\,(\operatorname{Im} c)\,\bar{\psi}\sigma^{\mu\nu}\gamma_5 \psi F_{\mu\nu} \equiv e\frac{a}{4m}\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu} + i\frac{d}{2}\bar{\psi}\sigma^{\mu\nu}\gamma_5\psi F_{\mu\nu} \; ,\end{aligned} \tag{1.31}$$

and define the $\mathcal{CP}$-violating **electric dipole moment** $d$ (EDM) and the $\mathcal{CP}$-conserving **magnetic anomalous moments** $a = (g-2)/2$ of the particle $\psi$. The former are extremely suppressed in the SM, while the latter are extremely well-measured for light leptons (for recent reviews, see e.g. Ref. [11, 40, 41]). Using the flavor-symmetry formalism, besides the fact that $d_\psi \sim m_\psi$ and $a \sim m_\psi^2$ from the left-right structure of the magnetic operator (see Eq. (1.29)), we can also predict the (weak) order at which the CKM phase can generate a quark or lepton EDM in the SM.

Let us start with the leptons and the $(E\mathbf{Y}_e \mathbf{X} \sigma_{\mu\nu} L) H^\dagger F^{\mu\nu}$ interaction with $\mathbf{X}$ some chains of spurion insertions. Since $\mathbf{Y}_e$ has a real background value, $\mathbf{X}$ must be proportional to a complex flavor

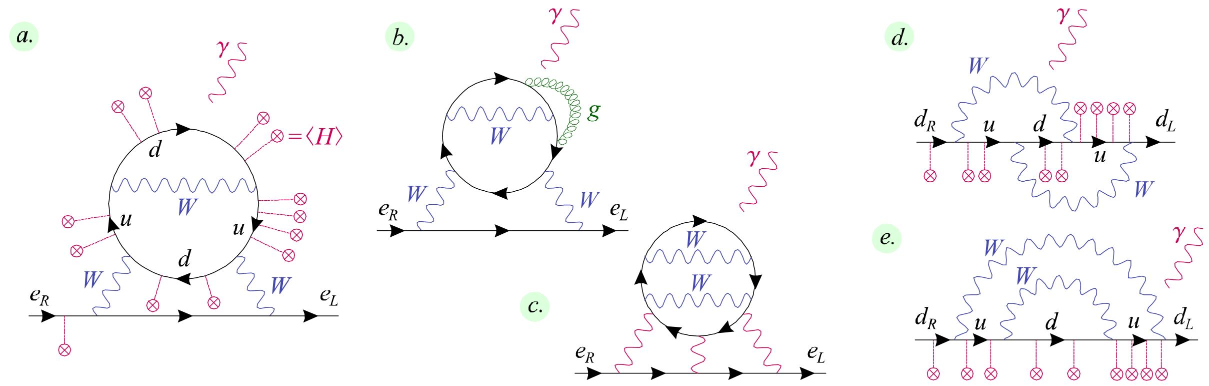


Figure 1.2: The CKM-induced EDM of leptons $(a - c)$ and quarks $(d - e)$ in the SM. For leptons (quarks), the three-loop (two-loop) electroweak contribution actually vanishes because of the anti-symmetry properties of the Jarlskog invariant. Four-loop processes are able to induce a non-zero $d_e$ through either QCD ($b$) or QED ($c$) corrections.

trace of a $G_F$-invariant chain of $\mathbf{Y}_u$ and $\mathbf{Y}_d$. The simplest such trace contains no less than twelve Yukawa insertions [23]

$$\begin{aligned}\mathbf{X} &= \langle(\mathbf{Y}_d^\dagger\mathbf{Y}_d)^2\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d(\mathbf{Y}_u^\dagger\mathbf{Y}_u)^2 - (\mathbf{Y}_d^\dagger\mathbf{Y}_d)^2(\mathbf{Y}_u^\dagger\mathbf{Y}_u)^2\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\rangle \\ &= 2i\,\mathrm{Im}\langle(\mathbf{Y}_d^\dagger\mathbf{Y}_d)^2\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d(\mathbf{Y}_u^\dagger\mathbf{Y}_u)^2\rangle = \det[\mathbf{Y}_u^\dagger\mathbf{Y}_u, \mathbf{Y}_d^\dagger\mathbf{Y}_d] \equiv 2iJ_{\mathcal{CP}}\ . \end{aligned} \tag{1.32}$$

The last equality follows from the Cayley-Hamilton theorem[5]. This quantity actually reduces to the very suppressed **Jarlskog invariant** [42]:

$$\mathbf{X} = 2i\mathcal{J}_{\mathcal{CP}} \times \prod_{\substack{i>j=d,s,b \\ i>j=u,c,t}} \frac{m_i^2 - m_j^2}{v^2} \approx 2i\mathcal{J}_{\mathcal{CP}} \times \frac{m_b^4 m_s^2 m_c^2}{v^8} \approx i \times 10^{-23}\ , \tag{1.33}$$

where $\mathcal{J}_{\mathcal{CP}} \approx A^2\lambda^6\eta \approx 3 \times 10^{-5}$. Note that $J_{\mathcal{CP}}$ vanishes if any two up or down-type quarks are degenerate, in a way reminiscent to the freedom one would get in that case to rotate the $\mathcal{CP}$-violating phase away.

This flavor structure tells us a number of things about the underlying electroweak process. First, it arises at least at three loops [43] since a closed quark loop with four $W$ boson vertices is required, see Fig. 1.2. Second, the flavors of the two up-type and two down-type quarks must be different otherwise the CKM factors combine into absolute values. Third, the amplitude must change sign under the exchange of either the two up or two down quarks to match the antisymmetric property of the Jarlskog invariant. But, naively, if we think of the $\mathbf{Y}_u$ and $\mathbf{Y}_d$ insertions as mass insertions along a closed quark loop, both terms entering in $\mathbf{X}$ would be generated without the minus sign. To break the symmetry of the mass insertions a further loop is required [43], e.g. a QCD correction.[6]

[5]To see this, it suffices to plug $\mathbf{X} = [\mathbf{Y}_u^\dagger\mathbf{Y}_u, \mathbf{Y}_d^\dagger\mathbf{Y}_d]$ in Eq. (A.30) of Appendix A.3, which simplifies greatly thanks to $\langle[\mathbf{Y}_u^\dagger\mathbf{Y}_u, \mathbf{Y}_d^\dagger\mathbf{Y}_d]\rangle = 0$. So, $\det[\mathbf{Y}_u^\dagger\mathbf{Y}_u, \mathbf{Y}_d^\dagger\mathbf{Y}_d]$ is non-zero only if there is a $\mathcal{CP}$ phase in $\mathbf{Y}_u^\dagger\mathbf{Y}_u$ and/or $\mathbf{Y}_d^\dagger\mathbf{Y}_d$.

[6]A word of caution about the mass-dependence: when using $G_F$, we are forced to work in the $SU(2)_L \otimes U(1)_Y$ invariant phase where fermions are massless, and spurion insertions are understood as Higgs tadpole insertions. They collapse to mass insertions after the SSB. Though this is fine to predict the flavor structure, some dynamical effects may be lost in such a perturbative treatment of the fermion masses. This actually happens for the photon penguin $sd \to \gamma^*$ discussed before. Because the massless amplitude is not safe in the infrared, the quadratic GIM breaking softens into a

Combining all this, we arrive at the rough estimate (see also Ref. [45]):

$$\frac{d_e}{e} \approx \frac{m_e}{M_W^2}\left(\frac{g^2}{16\pi^2}\right)^3 \frac{\alpha_S}{4\pi} J_{\mathcal{CP}} \approx 10^{-49} \; , \tag{1.34}$$

to be compared to the current limit $|d_e| < 8.7\cdot 10^{-29}$ $ecm$ (90%) [46]. The same quark loop drives the EDM of the $W$ boson (suffices to cut the two lower $W$ propagators in Fig. 1.2$a$), as well as those of the heavier leptons. Up to dynamical effects related to the different scales of these processes, we thus expect $d_e/m_e = d_\mu/m_\mu = d_\tau/m_\tau$, so $d_\mu$ and $d_\tau$ are about 200 and 4000 times larger than $d_e$. Though these estimates are not very precise, they all stand well beyond our reach experimentally, since the current limits are $|d_\mu| < 1.9\cdot 10^{-19}$ $ecm$ (95%) [47] and $d_\tau \in [-2.2, 4.5]\cdot 10^{-17}$ $ecm$ (95%) [48]. Those are far weaker than for $d_e$ which exploits the very high electric field present in the $ThO$ molecule. By contrast, the bound on $d_\mu$ was obtained alongside the precise $(g-2)_\mu$ measurement, and that for $d_\tau$ from the study of the $\gamma\tau^+\tau^-$ vertex using the $e^+e^- \to \tau^+\tau^-$ process at Belle.

Turning now to quarks, the generation of their EDM may look simpler at first sight since they can directly "feel" the CKM phase. However, the spurion technique easily shows that this not so in practice. Let us concentrate on the down quark, and thus on the interaction $(D\mathbf{Y}_d\mathbf{X}\sigma_{\mu\nu}Q)H^\dagger F^{\mu\nu}$ for some chains of spurions $\mathbf{X}$. Since we must pick $d_L$ in $Q$, we use the gauge basis in Eq. (1.21) and $\mathbf{Y}_d$ is diagonal. So, $\mathbf{X}$ must be a chain of $\mathbf{Y}_u^\dagger\mathbf{Y}_u$ and $\mathbf{Y}_d^\dagger\mathbf{Y}_d$, and its 1-1 entry needs to have a non-zero imaginary part to generate a $d$-quark EDM. But, with $\mathbf{Y}_d^\dagger\mathbf{Y}_d$ real and diagonal and $\mathbf{Y}_u^\dagger\mathbf{Y}_u$ hermitian, this requires again quite a long chain of spurions. The shortest turns out to be

$$\mathbf{X} = [\mathbf{Y}_u^\dagger\mathbf{Y}_u \; , \; \mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u] \; . \tag{1.35}$$

It needs to be antisymmetrized in this way because the sum of the two terms is hermitian, so with only real entries on the diagonal (besides being reducible via Cayley-Hamilton identities). The electroweak loops behind such a process thus share many of the features of those generating $d_e$. Two $W$ boson propagators are needed together with a further gluonic correction to break the symmetry of the mass insertions. The leading order thus arises at three loops, and has the generic form

$$\mathbf{X}^{11} = -2i\mathcal{J}_{\mathcal{CP}} \times \frac{m_b^2 - m_s^2}{v^2} \prod_{i>j=u,c,t} \frac{m_i^2 - m_j^2}{v^2} \to d_d \approx e\frac{m_d}{M_W^2}\left(\frac{g^2}{16\pi^2}\right)^2 \frac{\alpha_S}{4\pi}\frac{m_b^2 m_c^2}{v^4}\mathcal{J}_{\mathcal{CP}} \; , \tag{1.36}$$

leading to $d_d \approx 10^{-34} - 10^{-37}$ $ecm$ depending on the precise dependences on $m_b^2/v^2$ and $m_c^2/v^2$. Indeed, here also the mass-insertion approximation does not perfectly reproduce the explicit computation done in Ref. [49] , where the $m_b^2$ factor turns out to soften into a logarithmic GIM breaking. The prediction for $d_u$ is very similar though interchanging $\mathbf{Y}_d \leftrightarrow \mathbf{Y}_u$ in Eq. (1.35) leads to a further suppression by $m_s^2/m_b^2$. With this, the short-distance SM contribution to the EDM of the neutron $d_n \approx (4d_d - d_u)/3$ is predicted to be at most around $10^{-34}$ $e\cdot cm$. This is to be compared to the long-distance contributions which may enhance the SM contribution by up to two orders of magnitude to $d_n \approx 10^{-32}$ $ecm$ [50], and to the current bound which stands at $|d_n| < 2.9 \times 10^{-26}$ $ecm$ (90%) [51]. Note, finally, that because $\langle\mathbf{X}\rangle = 0$, we have the sum rule $d_d/m_d + d_s/m_s + d_b/m_b = 0$ up to kinematical effects beyond our control.

logarithmic breaking only (the function $F_\gamma(m_q^2/M_W^2)$ in Eq. (1.16) behaves as $\log m_q/M_W$ in the $m_q \to 0$ limit). The gluon penguin behaves similarly, and so does presumably the quark–W–gluon loop generating $d_e$. Though an explicit computation of this four-loop amplitude has not been done yet, such an effect was found for the similar CKM-induced triple gluon $\mathcal{CP}$-violating operator, $f^{abc}\tilde{G}^a_{\mu\nu}G^{b,\nu\rho}G^c_{\rho\mu}$ [44]. There, heavy quark factors like the two $m_b^2/v^2$ suppression factors in $\mathbf{X}$ get replaced by logarithmic factors of ratios of masses. For this reasons, up to a few orders of magnitude enhancement are understood for estimates like Eq. (1.34).

# Chapter 2

# MFV: Purpose, definition, application

## 2.1 The New Physics flavor puzzles

The quark flavor structures have been probed extensively through many dedicated experiments. No significant deviation with respect to the SM has ever been seen. In particular, the CKM pattern of flavor mixing and $\mathcal{CP}$-violation is very well supported, as summarized by the well-known unitary triangle plot [36] shown in Fig. 2.1. New Physics effects, if present, must be small and preserve all these patterns to an excellent approximation. They should upset neither the suppression of FCNC nor their hierarchies. Our goal in this section is to make this statement more precise, in order to really appreciate the level of precision and/or the tightness of the constraints derived from flavor observables.

To this end, imagine that NP particles all have mass greater than the SM energy scale $v = 174$ GeV. These particles are thus never produced on-shell, but their virtual effects are nevertheless felt at low energy. They can take three different forms [52, 53]:

1. Virtual exchange of the new particles could induce interactions that are similar to those already present in the SM. In that case, their impact sums up to mere shifts in the values of the SM free parameters. But since those parameters have to be fixed at their measured values anyway, there is no way to tell if NP is present or not.

2. The virtual effects could induce new interactions among SM particles. Since the SM contains all the renormalizable interactions compatible with the prescribed symmetries, these interactions can be encoded into nonrenromalizable but gauge invariant operators,

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_{i,d>4} \frac{\mathcal{C}_i}{\Lambda^{d-4}} \mathcal{O}_i^d \,, \tag{2.1}$$

   where $d$ is the (mass) dimension of the **effective operator** $\mathcal{O}_i$, $\Lambda$ the scale of NP (the typical mass of the new particles), and $\mathcal{C}_i$ some numerical constants called the **Wilson coefficients**. Constraints on the processes to which each $\mathcal{O}_i^d$ contribute then translate as constraints on the mass scale of the NP, up to some assumptions on the values of the $\mathcal{C}_i$. For $\Lambda$ greater than both the electroweak scale $v$ and the typical energy of the processes under consideration, only the lowest-dimensional effective operators are relevant.

3. The first two cases arise automatically whenever the SM emerges as a well-defined low-energy limit of the NP theory. This means that the SM symmetries are not explicitly broken by the NP dynamics, and all the new particles are decoupled from those of the SM. For example, scenarios where the photon has a tiny mass or the Higgs boson is embedded into a $SU(2)_L$

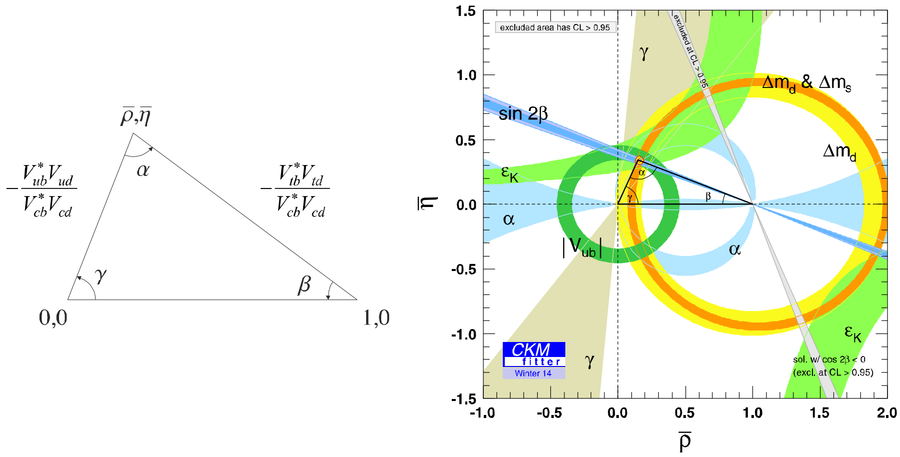


Figure 2.1: The definition of the unitary triangle, and its determination from $B$ and $K$ physics observables [36]. All the current experimental information agree. In particular, a unique source for all CP-violation ($\eta \neq 0$) is strongly favored.

triplet instead of a doublet are not immediately covered by this formalism. We will not consider these alternatives here.

A full analysis of the constraints on all the higher-dimensional operators would drive us far beyond our main topic of interests, so let us focus on the few of them we encountered in the previous chapter: the $Z$ penguin, the $W$ box, and the magnetic penguin operators:

$$\mathcal{H}_{eff}^{NP} = \frac{\mathcal{C}_Z^{IJ}}{\Lambda^2}(\bar{Q}^I\gamma_\mu Q^J)H^\dagger D_\mu H + \frac{\mathcal{C}_{WW}^{IJ}}{\Lambda^2}(\bar{Q}^I\gamma_\mu Q^J)(\bar{Q}^I\gamma^\mu Q^J) + \frac{e\mathcal{C}_\gamma^{IJ}}{\Lambda^2}D^I\sigma_{\mu\nu}Q^J F^{\mu\nu}H^C + h.c.\;. \tag{2.2}$$

To derive constraints on the scale $\Lambda$, three different assumptions are made on the Wilson coefficients. First, as is customary in physics, naturalness would demand $\mathcal{C}_i \sim \mathcal{O}(1)$. In other words, lacking any precise dynamical information, dimensional analysis alone is supposed to catch the right order of magnitude of the effects. A second possibility, since in many NP models the FCNC still arise at the loop level, is to set the Wilson coefficients at $(\mathcal{C}_{WW}^{IJ})_{\rm NP} \sim (\mathcal{C}_{Z,\gamma}^{IJ})_{\rm NP}^2 \sim \mathcal{O}((g^2/4\pi)^2)$, with $g$ the weak coupling constant. In the third situation, we assume that the CKM scaling is preserved by the NP dynamics, in which case $|\mathcal{C}_{Z,\gamma}^{IJ}|_{\rm NP} \sim |V_{tI}^*V_{tJ}|$ and $|\mathcal{C}_{WW}^{IJ}|_{\rm NP} \sim |V_{tI}^*V_{tJ}|^2$ with

$$|V_{tb}^*V_{ts}| \approx 4\cdot 10^{-2}\;,\; |V_{tb}^*V_{td}| \approx 8\cdot 10^{-3},\; |V_{ts}^*V_{td}| \approx 3\cdot 10^{-4}\;. \tag{2.3}$$

A final assumption is to take $|\mathcal{C}_{Z,\gamma}^{IJ}|_{\rm NP} \sim |V_{tI}^*V_{tJ}| \times g^2/4\pi$ and $|\mathcal{C}_{WW}^{IJ}|_{\rm NP} \sim |V_{tI}^*V_{tJ}|^2 \times (g^2/4\pi)^2$, which are the naive scaling of the SM contributions. One can then check that the experimental data ask for $\Lambda$ around the electroweak scale.

Let us start with the meson mixing $B_s^0 - \bar{B}_s^0$, $B_d^0 - \bar{B}_d^0$, and $K^0 - \bar{K}^0$. Details of the evaluations are in the Appendix A.1 (see also [54]), and we here simply quote the approximate bound on the scale $\Lambda$:

| $(\mathcal{C}_{WW}^{IJ})_{\rm NP}$ | $\mathcal{O}(1)$ | $\mathcal{O}((g^2/4\pi)^2)$ | $\mathcal{O}(\lvert V_{tI}^*V_{tJ}\rvert^2)$ | $\mathcal{O}(\lvert V_{tI}^*V_{tJ}\rvert^2 \times (g^2/4\pi)^2)$ |
|---|---|---|---|---|
| $B_s^0 - \bar{B}_s^0$ | $\Lambda \gtrsim 130$ TeV | $\Lambda \gtrsim 4$ TeV | $\Lambda \gtrsim 5$ TeV | $\Lambda \gtrsim 0.17$ TeV |
| $B_d^0 - \bar{B}_d^0$ | $\Lambda \gtrsim 650$ TeV | $\Lambda \gtrsim 21$ TeV | $\Lambda \gtrsim 5$ TeV | $\Lambda \gtrsim 0.16$ TeV |
| $K^0 - \bar{K}^0$ | $\Lambda \gtrsim 24000$ TeV | $\Lambda \gtrsim 800$ TeV | $\Lambda \gtrsim 8$ TeV | $\Lambda \gtrsim 0.25$ TeV |

(2.4)

The strongest constraints on $\Lambda$ clearly come from the kaon sector. This can be simply understood. The experimental results are in good agreement with the SM, so they roughly scale like the corresponding SM contributions. The kaon sector is the most suppressed by the CKM scaling, hence it is the one leaving the least room for NP[1].

For the $Z$ penguin, the cleanest constraints come from leptonic and semileptonic operators because the hadronic matrix elements are well-controlled theoretically. The golden modes, for which to an excellent approximation only the $Z$ penguin contributes, are the $B_{d,s} \to \mu^+\mu^-$ decays, along with $K^+ \to \pi^+\nu\bar{\nu}$, from which

| $(\mathcal{C}_Z^{IJ})_{\rm NP}$ | $\mathcal{O}(1)$ | $\mathcal{O}(g^2/4\pi)$ | $\mathcal{O}(\lvert V_{tI}^* V_{tJ}\rvert)$ | $\mathcal{O}(\lvert V_{tI}^* V_{tJ}\rvert \times g^2/4\pi)$ |
|---|---|---|---|---|
| $B_s \to \mu^+\mu^-$ | $\Lambda \gtrsim 12$ TeV | $\Lambda \gtrsim 2.2$ TeV | $\Lambda \gtrsim 2.5$ TeV | $\Lambda \gtrsim 0.45$ TeV |
| $B_d \to \mu^+\mu^-$ | $\Lambda \gtrsim 17$ TeV | $\Lambda \gtrsim 3$ TeV | $\Lambda \gtrsim 1.5$ TeV | $\Lambda \gtrsim 0.27$ TeV |
| $K^+ \to \pi^+\nu\bar{\nu}$ | $\Lambda \gtrsim 100$ TeV | $\Lambda \gtrsim 18$ TeV | $\Lambda \gtrsim 1.8$ TeV | $\Lambda \gtrsim 0.33$ TeV |

(2.5)

Once again, $K$ physics is the most constraining for generic Wilson coefficients, while $B_s \to \mu^+\mu^-$ becomes better when these scale as in the SM. In principle, the $B_{d,s} \to (K,\pi,...)\nu\bar{\nu}$ or $B_{d,s} \to (K,\pi,...)\ell^+\ell^-$ processes could also be used (see e.g. Ref. [55]). In particular, recent LHCb results [56] on the $B \to K^*\ell^+\ell^-$ angular analysis and on lepton universality test $B^+ \to K^+\ell^+\ell^-$, $\ell = e,\mu$, slightly off their expected values in the SM, have generated a lot of activity [57], but discussing this in details would drive us too far afield. In the kaon sector, the $K_L \to \mu^+\mu^-$ decay is also very well measured but cannot be used to draw competitive constrains on the $Z$ penguin. This process mostly proceeds through $K_L \to \gamma\gamma \to \mu^+\mu^-$, dominated by long-distance QCD effects and plagued by large theoretical uncertainties [58].

For the magnetic operators, the analysis proceeds along similar lines, except for the presence of the chirality-flip factor. In principle, the NP dynamics could induce this flip differently, thereby bypassing the strong $m_b/v$ or $m_s/v$ suppression. The physically sensible scalings for the Wilson coefficients and the corresponding scales are then[2]

| $(\mathcal{C}_\gamma^{I\neq J})_{\rm NP}$ | $\mathcal{O}(1)$ | $\mathcal{O}(m_{b,s}/v)$ | $\mathcal{O}(g^2/4\pi \times m_{b,s}/v)$ | $\mathcal{O}(\lvert V_{tI}^* V_{tJ}\rvert \times m_{b,s}/v)$ |
|---|---|---|---|---|
| $b \to s\gamma$ | $\Lambda \gtrsim 220$ TeV | $\Lambda \gtrsim 34$ TeV | $\Lambda \gtrsim 6$ TeV | $\Lambda \gtrsim 7$ TeV |
| $b \to d\gamma$ | $\Lambda \gtrsim 56$ TeV | $\Lambda \gtrsim 9$ TeV | $\Lambda \gtrsim 1.5$ TeV | $\Lambda \gtrsim 0.8$ TeV |
| $s \to d\gamma$ | $\Lambda \gtrsim 220$ TeV | $\Lambda \gtrsim 5$ TeV | $\Lambda \gtrsim 0.9$ TeV | $\Lambda \gtrsim 0.1$ TeV |

(2.6)

The high sensitivity of $b \to s\gamma$ makes it one of the golden modes for $B$ physics. In the $K$ sector, the bounds are hardly competitive because the experimental sensitivity is not sufficient to reach the SM level [59]. Note however that if magnetic and chromomagnetic penguin operators like $\mathcal{C}_g^{IJ} D^I \sigma_{\mu\nu} T^a Q^J G_a^{\mu\nu} H^C$ are related, then the measured $\varepsilon'_K$ would imply $\Lambda \gtrsim 4000$ TeV when $\mathcal{C}_g^{21}$ is $\mathcal{O}(1)$, far above the corresponding bounds in the $B$ sector.

For comparison, we can repeat this exercise with the leptonic magnetic operators $E^I \sigma_{\mu\nu} L^J F^{\mu\nu} H^C$, which are forbidden in the SM. For instance, the quite stringent experimental bounds, $\mathcal{B}(\mu \to e\gamma)^{\rm exp} < 5.7 \times 10^{-13}$, $\mathcal{B}(\tau \to e\gamma)^{\rm exp} < 3.3 \times 10^{-8}$, and $\mathcal{B}(\tau \to \mu\gamma)^{\rm exp} < 4.4 \times 10^{-8}$ [17] imply

| $(\mathcal{C}_\gamma^{I\neq J})_{\rm NP}$ | $\mathcal{O}(1)$ | $\mathcal{O}(m_{\mu,\tau}/v)$ | $\mathcal{O}(m_\nu/v)$ |
|---|---|---|---|
| $\mu \to e\gamma$ | $\Lambda \gtrsim 25000$ TeV | $\Lambda \gtrsim 610$ TeV | $\Lambda \gtrsim 0.1$ TeV |
| $\tau \to (e,\mu)\gamma$ | $\Lambda \gtrsim 250$ TeV | $\Lambda \gtrsim 26$ TeV | $\Lambda \gtrsim 0.001$ TeV |

(2.7)

[1]Meson mixings actually probe NP up to slightly higher scales, but through other non-standard operators. For example, the contribution to $K^0 - \bar{K}^0$ of a scalar operator like $(\bar{s}_R d_L)(\bar{s}_L d_R)$ is enhanced by both its QCD evolution down to the hadronic scale and its matrix element. If its Wilson coefficient is of $\mathcal{O}(1)$, $\Lambda$ is pushed well above $10^5$ TeV.

[2]In this case, the SM contribution is not neglected, so we do not include the case where $C_\gamma^{IJ}$ scales like in the SM. See Appendix A.1 for more details.

The second column corresponds to the usual helicity suppression, i.e., one chirality flip on the external heavy lepton line. This is hardly sufficient to make the scale $\Lambda$ accessible to the LHC. So for the third case, we take the extreme assumption of setting the Wilson coefficients proportional to neutrino masses $m_\nu \sim 1$ eV. Indeed, we have argued that LFV can only occur once $m_\nu \neq 0$. By analogy with the down quark FCNC, which are driven by the GIM breaking induced by the mass splitting of up-type quarks, the charged lepton FCNC could be induced by neutrino mass splitting. This is the situation in the SM supplemented with Dirac neutrino masses. Because the experimental reach is then very far from the theoretical predictions, the scale $\Lambda$ ends up extremely low, even below the range permitting an effective treatment (which asks for $\Lambda/v > 1$).

The magnetic operators also induce EDM when the Wilson coefficients are complex. For example $\operatorname{Im}\mathcal{C}_\gamma^{11}$ induces $d$-quark EDM, which then generate a neutron EDM. Using the approximation $d_n \approx (4d_d - d_u)/3$ [11], the current bound $|d_n| < 2.9 \times 10^{-26}$ $ecm$ (90%) [51] implies

| $(\mathcal{C}_\gamma^{II})_{\rm NP}$ | $\mathcal{O}(1)$ | $\mathcal{O}(m_\psi/v)$ | $\mathcal{O}(m_\psi/v \times \mathcal{J}_{\mathcal{CP}})$ |
|---|---|---|---|
| $d_n$ | $\Lambda \gtrsim 15000$ TeV | $\Lambda \gtrsim 120$ TeV | $\Lambda \gtrsim 0.6$ TeV |
| $d_e$ | $\Lambda \gtrsim 280000$ TeV | $\Lambda \gtrsim 480$ TeV | $\Lambda \gtrsim 2.6$ TeV |
| $d_{Hg}$ | $\Lambda \gtrsim 39000$ TeV | $\Lambda \gtrsim 300$ TeV | $\Lambda \gtrsim 1.6$ TeV |
| $a_\mu$ | $\Lambda \gtrsim 350$ TeV | $\Lambda \gtrsim 8.7$ TeV | – |

(2.8)

For comparison, we have included the corresponding bound from the EDM of the electron , $d_e < 8.7 \times 10^{-29}$ $ecm$ [46], and mercury, $d_{Hg} < 3.1 \times 10^{-29}$ $ecm$ [60]. In the latter case, a theoretical assumption is made because $d_{Hg}$ is sensitive mainly on the $\mathcal{CP}$-violation occurring in the $\pi NN$ couplings, which can be induced by the chromomagnetic operators (see Appendix A.1). The bounds in the above table are drawn assuming the same NP dynamics generate both the electro- and chromomagnetic operators (typically, these operators arise at the loop level and some of the virtual NP particles circulating in the loop are necessarily electrically charged and colored). The three assumptions on the Wilson coefficients actually hold on $\operatorname{Im}\mathcal{C}_\gamma^{II}$ for EDMs, and on $\operatorname{Re}\mathcal{C}_\gamma^{\mu\mu}$ for the magnetic anomalous moment $a_\mu$ (see Appendix A.1). For the last two columns, $m_\psi$ denotes the mass of the fermion involved, i.e., $m_d$ for $n$ and $H_g$, $m_{e,\mu}$ for $d_e$ and $a_\mu$, respectively.

The results of this section strikingly illustrate the **flavor puzzle**: the NP contributions to the FCNC are compatible with the experimental constraints when either the NP scale is very high, or the NP flavor structures are far from generic. In both cases, naturality is in danger. A very heavy NP models necessitates delicate fine-tuning of its parameters in order to maintain the large splitting with the electroweak scale. With our current understanding, this **hierarchy puzzle** should be avoided at all cost, so some NP should kick in not too far from the TeV scale. In that case, the NP flavor sector must necessarily be at least as peculiar as that of the SM, with for example a very strong suppression of the $s \to d$ transitions. In general, this requires fine-tuning many of the flavored couplings of the NP models [18]. It is the purpose of the next section to quantify the extent of these fine-tunings.

The situation is different for flavor-blind $\mathcal{CP}$-violating observables like EDMs, giving rise to the so called $\mathcal{CP}$ **puzzle** [11]. At first glance, it appears that even if $\mathcal{CP}$-violation is forced to somehow arise from an SM-like flavor sector, so that it get suppressed by $\mathcal{J}_{\mathcal{CP}} \sim 10^{-5}$, the required NP scale remains slightly too high. The specific dynamics of some NP models could permit to bring down this scale a bit, but the situation is not very comfortable because the suppression by $\mathcal{J}_{\mathcal{CP}} \sim 10^{-5}$ is questionable. The intimate connection between $\mathcal{CP}$-violation and flavor transitions is a peculiarity of the CKM paradigm, but it does not rest on firm ground. A priori, any Lagrangian parameter can be complex hence $\mathcal{CP}$ violating. Even though many of these phases can be absorbed into redefinitions of the fields and a global phase of the Lagrangian is irrelevant, there is no guarantee this suffices to get rid of all the $\mathcal{CP}$-violating phases in the non-flavored sectors. This already occurs in the SM: the strong $\theta$ parameter arises in the QCD Lagrangian, and induce $\mathcal{CP}$-violating effects in the strong interaction. To pass the EDM constraints, this parameter has to be fine-tuned, $\theta \lesssim 10^{-10}$, since the scale of the SM is fixed at $\Lambda \approx 100$ GeV. This is called the **strong $\mathcal{CP}$ puzzle** of the SM [12]. In most models of NP, this situation worsens as many additional flavor-blind parameters are allowed to be complex. So, either the NP scale is huge, or these new phases are as fine-tuned as the SM $\theta$ parameter.

## 2.2 Introducing Minimal Flavor Violation

For most operators, the Wilson coefficients have to be severely suppressed if the NP scale is to be only slightly above the electroweak scale. But, actually, there is no way to tell whether these suppressions are natural or not. Indeed, the naive definition of naturality –Lagrangian parameters should be $\mathcal{O}(1)$– makes no sense in the flavor sector, where the known Yukawa couplings are already highly non-natural. After all, $m_u$ is a tiny number compared to $m_t$. So, the best strategy to define a meaningful naturality principle for the NP flavor couplings is to compare them with the Yukawa couplings. There would be no flavor puzzle if the hierarchies of the NP flavor couplings required to pass the experimental constraints are similar to those observed for the quark and lepton masses and mixings. In other words, the NP flavor sector would then be as fine-tuned, and thus no less natural, than that of the SM.

To proceed, this similarity statement must be made precise. We will ground it on a symmetry principle using $G_F$, and deem natural those NP flavor couplings which respect **Minimal Flavor Violation**. Let us thus turn to the two-step definition of this hypothesis [23]. The first specifies how the flavor couplings are to be constructed, and the second requires the free parameters to be natural.

### 2.2.1 Construction principle

The first condition for MFV is expressed straightforwardly in the spurion language: all the flavor couplings are required to be invariant under $G_F$, but only the spurions $\mathbf{Y}_{u,d,e}$ needed to account for the fermion masses and mixings are allowed. This is clearly a minimal breaking of $G_F$, since anything less would be insufficient to reproduce the well-known fermionic flavor structures. Typically, this does not forbid NP from introducing new flavor couplings, but forces them to be expressed as polynomials in the allowed spurions, that is, as functions of the Yukawa couplings $\mathbf{Y}_{u,d,e}$.

Let us take again the penguin operator as an example, and specifically, the two currents:

$$\mathcal{H}_{eff}^{NP} = \frac{\mathcal{C}_{ZLL}^{IJ}}{\Lambda^2}(\bar{Q}^I\gamma_\mu Q^J)H^\dagger D_\mu H + \frac{\mathcal{C}_{ZRR}^{IJ}}{\Lambda^2}(D^I\gamma_\mu \bar{D}^J)H^\dagger D_\mu H \ . \tag{2.9}$$

The Wilson coefficients $\mathcal{C}_{ZLL}^{IJ}$ and $\mathcal{C}_{ZRR}^{IJ}$ are three-by-three matrices of complex numbers in flavor space which explicitly breaks the $G_F$ symmetry. Whenever $\mathcal{C}_{ZLL}, \mathcal{C}_{ZRR} \neq \mathbf{1}$, these matrices are not invariant under $Q \to g_Q Q$, $D \to D g_D$; the values of their entries depend on the basis chosen for the quark fields.

To formally restore the $G_F$ invariance, $\mathcal{C}_{ZLL}$ and $\mathcal{C}_{ZRR}$ must transform contragradiantly to the fields, that is, $\mathcal{C}_{ZLL} \to g_Q \mathcal{C}_{ZLL} g_Q^\dagger$ and $\mathcal{C}_{ZRR} \to g_D^\dagger \mathcal{C}_{ZRR} g_D$. This can be achieved thanks to the presence of the spurions. There are infinitely many combinations of spurions transforming like that. So in full generality, the flavor couplings are written as expansions

$$\mathcal{C}_{ZLL} = z_1^{LL}\mathbf{1} + z_2^{LL}\mathbf{Y}_u^\dagger\mathbf{Y}_u + z_3^{LL}\mathbf{Y}_d^\dagger\mathbf{Y}_d + z_4^{LL}\{\mathbf{Y}_d^\dagger\mathbf{Y}_d, \mathbf{Y}_u^\dagger\mathbf{Y}_u\} + \ldots \ , \tag{2.10}$$

$$\mathcal{C}_{ZRR} = z_1^{RR}\mathbf{1} + z_2^{RR}\mathbf{Y}_d\mathbf{Y}_d^\dagger + z_3^{RR}\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger + z_4^{RR}\mathbf{Y}_d\mathbf{Y}_u^\dagger\mathbf{Y}_u\mathbf{Y}_d^\dagger + \ldots \ , \tag{2.11}$$

for some a priori complex coefficients $z_i^{LL,RR}$. Clearly, under a $G_F$ transformation, $\mathcal{C}_{ZLL} \to g_Q \mathcal{C}_{ZLL} g_Q^\dagger$ and $\mathcal{C}_{ZRR} \to g_D^\dagger \mathcal{C}_{ZRR} g_D$ are ensured entirely by the transformation rules of the Yukawa spurions, see Eq. (1.20).

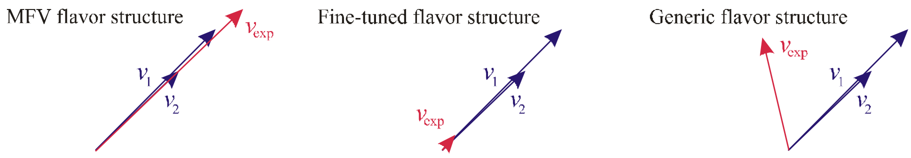


Figure 2.2: The three possible situation for the NP flavor structures. The vectors $v_{1,2}$ represent the basis made of the $G_F$ symmetric terms of (2.10). Expressed in this basis, $v_{\rm exp}$, obtained from flavor experiments, requires either $\mathcal{O}(1)$, $\ll 1$, or $\gg 1$ coefficients.

At this stage, we have not achieved much yet. Indeed, once this expansion is written down, the spurions have to be frozen back to their physical values in some basis. For example, we can now set $v\mathbf{Y}_u \sim \mathbf{m}_u V_{CKM}$, $v\mathbf{Y}_d \sim \mathbf{m}_d$. But, without any constraint on the $z_i$, any coupling can be expressed in this way. Said differently, the infinite series of powers of $\mathbf{Y}_u^\dagger \mathbf{Y}_u$ and $\mathbf{Y}_d^\dagger \mathbf{Y}_d$ form a complete basis for the space of complex three-by-three matrices [22]. So, the flavor couplings $\mathcal{C}_{ZLL}$ and $\mathcal{C}_{ZRR}$ can still be fully generic, i.e., with all their entries of $\mathcal{O}(1)$, and the flavor puzzle is not alleviated in any way.

### 2.2.2 Naturality principle

The spurion expansions are not entirely void of physical content. The numerical value of a flavor coupling like $\mathcal{C}_{ZLL}^{IJ}$ depends on the basis chosen for the quark fields. For example, its value in the $\mathbf{Y}_u \sim \mathbf{m}_u V_{CKM}$, $\mathbf{Y}_d \sim \mathbf{m}_d$ basis is different than that in the $\mathbf{Y}_u \sim \mathbf{m}_u$, $\mathbf{Y}_d \sim \mathbf{m}_d V_{CKM}^\dagger$ basis, with the two related as $\mathcal{C}_{ZLL} \to V_{CKM} \mathcal{C}_{ZLL} V_{CKM}^\dagger$. This is not very convenient in practice, because it renders any assertion on the size of the NP flavor couplings ambiguous. On the other hand, by construction, the coefficients occurring in the spurion expansions do not depend on the basis chosen for the quark fields [23]. So, they offer an unambiguous parametrization of the new flavor couplings. In particular, the experimental information drawn from flavor observables can be unambiguously translated into values or bounds for the coefficients of the spurion expansions. Three situations can arise, see Fig. 2.2:

- *MFV flavor structure*: The second condition for MFV is for all coefficients to be natural, $z_i \sim \mathcal{O}(1)$. In that case, all the flavor couplings inherit the hierarchies of the spurions. For example, the leading non-diagonal effects for $\mathcal{C}_{ZLL}$ arise from

$$\mathcal{C}_{ZLL}^{I\neq J} \approx z_2^{LL} \, (\mathbf{Y}_u^\dagger \mathbf{Y}_u)^{IJ} \approx z_2^{LL} \, (m_t^2/v^2) \; V_{3I}^* V_{3J} \; , \tag{2.12}$$

$$\mathcal{C}_{ZRR}^{I\neq J} \approx z_3^{RR} \, (\mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u \mathbf{Y}_d^\dagger)^{IJ} \approx z_3^{RR} \, (m_d^I m_d^J/v^2)(m_t^2/v^2) \; V_{3I}^* V_{3J} \; , \tag{2.13}$$

  in the basis in which $\mathbf{Y}_u \sim \mathbf{m}_u V_{CKM}$, $\mathbf{Y}_d \sim \mathbf{m}_d$, adequate to deal with operators involving down-type quarks. These are precisely the CKM coefficients and chiral suppression factors occurring for the SM transitions, see Eq. (1.26). So, in this case, we have achieved our goal to pass the SM hierarchies onto the NP flavor couplings. If this suppression is necessary and sufficient for all FCNC processes, MFV solves the flavor puzzles [20, 61]. This seems to be the case currently, as can be seen looking e.g. at Eq. (2.5).

- *Fine-tuned flavor structure*: Some coefficients are still required to be very small, $z_i \ll 1$. The suppression brought in by MFV is not sufficient, i.e. MFV fails to solve at least one flavor puzzle. In this case, one needs either a NP scale much higher than the TeV, or a complementary/alternative fine-tuning mechanism for that specific flavor coupling. As discussed in Sec. 2.2.4, this is partly the case for flavor-blind $\mathcal{CP}$-violating effects.

- *Generic flavor structure*: Some coefficients are required to be very large, $z_i \gg 1$, for instance if some FCNC processes deviate too much from their SM values. This would signal NP, of course, but also the presence of a new flavor structure within its dynamics. Indeed, though the terms of the expansions (2.10) form a complete basis, they barely do so; they nearly live in a lower-dimensional subspace. Therefore, a flavor structure not sufficiently aligned with $\mathbf{Y}_{u,d,e}$ generates huge coefficients when projected onto the expansions (2.10).

MFV thus offers an unambiguous test of naturalness. It permits to precisely characterize the flavor puzzles and to identify non-standard flavor structures.

### 2.2.3 Expansions systematics and properties

All the MFV expansions can be reduced to a finite number of terms thanks to simple algebraic manipulations [21]. To illustrate this, let us consider a generic operator $\mathbf{X}_8$ transforming as an octet under $U(3)_Q$. In full generality, it can be parametrized as an infinite series of products of powers of the hermitian matrices $\mathbf{X}_{u,d} \equiv \mathbf{Y}^\dagger_{u,d}\mathbf{Y}_{u,d}$ also transforming as octets under $U(3)_Q$,

$$\mathbf{X}_8 = \sum_{i,j,k,\ldots=0,1,2,\ldots} z_{ijk\ldots}\mathbf{X}_u^i\mathbf{X}_d^j\mathbf{X}_u^k\ldots\ , \tag{2.14}$$

for some appropriate coefficients $z_{ijk\ldots}$, a priori all complex. This series can be partially resummed using the Cayley-Hamilton identities (see Appendix A.3), which permit to express higher powers of any matrices in terms of its lower powers, traces, and determinant. For example, a term like $\mathbf{X}_u^3$ can be absorbed into redefinitions of the $z$, $z_1$, and $z_2$ coefficients using Eq. (A.29). Importantly, this reduction never generates large numerical coefficients because the traces satisfy $\langle\mathbf{X}_{u,d}\rangle \lesssim \mathcal{O}(1)$. It thus preserves the MFV naturalness principle. This leaves the octet operator $\mathbf{X}_8$ with 17 terms:

$$\begin{aligned}\mathbf{X}_8 = {}& x_1\mathbf{1} + x_2\mathbf{X}_u + x_3\mathbf{X}_d + x_4\mathbf{X}_u^2 + x_5\mathbf{X}_d^2 + x_6\{\mathbf{X}_u,\mathbf{X}_d\} + x_7 i[\mathbf{X}_u,\mathbf{X}_d] + x_8\mathbf{X}_u\mathbf{X}_d\mathbf{X}_u \\ &+ x_9 i[\mathbf{X}_d,\mathbf{X}_u^2] + x_{10}\mathbf{X}_d\mathbf{X}_u\mathbf{X}_d + x_{11} i[\mathbf{X}_u,\mathbf{X}_d^2] + x_{12}\mathbf{X}_d\mathbf{X}_u^2\mathbf{X}_d + x_{13} i[\mathbf{X}_u^2,\mathbf{X}_d^2] \\ &+ x_{14} i(\mathbf{X}_u\mathbf{X}_d\mathbf{X}_u^2 - \mathbf{X}_u^2\mathbf{X}_d\mathbf{X}_u) + x_{15} i(\mathbf{X}_d^2\mathbf{X}_u\mathbf{X}_d - \mathbf{X}_d\mathbf{X}_u\mathbf{X}_d^2) \\ &+ x_{16} i(\mathbf{X}_u\mathbf{X}_d^2\mathbf{X}_u^2 - \mathbf{X}_u^2\mathbf{X}_d^2\mathbf{X}_u) + x_{17} i(\mathbf{X}_d^2\mathbf{X}_u^2\mathbf{X}_d - \mathbf{X}_d\mathbf{X}_u^2\mathbf{X}_d^2)\ .\end{aligned} \tag{2.15}$$

The only non-trivial reduction is that for the term $\mathbf{X}_d^2\mathbf{X}_u\mathbf{X}_d\mathbf{X}_u^2$, which can be achieved by plugging $\mathbf{X} = [\mathbf{X}_u,\mathbf{X}_d]$ in Eq. (A.29). Also, we have used the hermiticity of $\mathbf{X}_{u,d}$ to write $\mathbf{X}_8$ entirely in terms of independent hermitian combinations of spurions [23].

The algebraic reduction leaves more than nine terms, even though this would be sufficient to span the whole vector space of complex matrices $\mathbf{X}_8$. So, we can further remove eight terms. But in doing so by hand, we must be careful to preserve the MFV scaling. Imagine for example that we remove the leading term $\mathbf{1}$. Then, the other terms of the expansion would need large coefficients to reproduce it, essentially because $\mathbf{X}_u$ and $\mathbf{X}_d$ have tiny 1-1 entries.

To proceed, let us first note that an additional approximate reduction can be performed using the fact that quark masses are highly hierarchical. Third-generation dominance expresses itself as $v^2\mathbf{X}_{u,d}^2 \simeq m_{t,b}^2\mathbf{X}_{u,d}$ and is valid to an excellent approximation. Imposing this as an exact identity, only four unaligned combinations of spurions need to be kept:

$$\mathbf{X}_8 \simeq x_1\mathbf{1} + x_2\mathbf{X}_u + x_3\mathbf{X}_d + x_6\{\mathbf{X}_u,\mathbf{X}_d\} + x_7 i[\mathbf{X}_u,\mathbf{X}_d]\ . \tag{2.16}$$

To get a MFV-compatible complete basis, at least these five terms must be present. The other terms are less important numerically, and we can take for example

$$\begin{aligned}\mathbf{X}_8 = {}& a_1\mathbf{1} + a_2\mathbf{X}_u + a_3\mathbf{X}_d + a_4\mathbf{X}_u^2 + a_5\mathbf{X}_d^2 + a_6\{\mathbf{X}_u,\mathbf{X}_d\} \\ &+ b_1 i[\mathbf{X}_u,\mathbf{X}_d] + b_2 i[\mathbf{X}_d,\mathbf{X}_u^2] + b_3 i[\mathbf{X}_u,\mathbf{X}_d^2]\ .\end{aligned} \tag{2.17}$$

This is the final form we shall use in the following. It is particularly convenient both when $\mathbf{X}_8$ is a generic complex coupling, or when it is hermitian. In this latter case, the expansion is the same but the nine coefficients have to be real. We keep it symmetrical in $\mathbf{X}_u$ and $\mathbf{X}_d$ and do not use the fact that $\mathbf{X}_d \ll \mathbf{X}_u$ because this is only true in the SM. For example, if the masses of up- and down-type quarks are tuned by different Higgs doublets, as in the MSSM, one could have $\mathbf{X}_d \sim \mathcal{O}(\mathbf{X}_u)$.

### 2.2.4 $\mathcal{CP}$-violating sources

Up to now, MFV was constructed to deal with flavor-violating transitions, but what about $\mathcal{CP}$-violating flavor-diagonal observables like EDM? There are two possible sources of $\mathcal{CP}$-violation: the spurions which involve a $\mathcal{CP}$-violating parameter (the CKM matrix is hidden there), and the expansion coefficients themselves since they are complex in general[3]. There is a convenient and phenomenologically oriented classification of these $\mathcal{CP}$-violating phases, based on their effects on flavor-blind observables like EDM [23]. Let us thus consider the magnetic operator, and adopt a generic MFV expansion for its Wilson coefficient:

$$\mathcal{H}_{eff} = \frac{e\mathcal{C}_\gamma^{IJ}}{\Lambda^2} D^I \sigma_{\mu\nu} Q^J F^{\mu\nu} H^C \ , \ \mathcal{C}_\gamma = \mathbf{Y}_d \mathbf{X}_8 \ , \tag{2.18}$$

where $\mathbf{X}_8$ introduces nine free complex parameters, see Eq. (2.17). Following Ref. [23] (see also Ref. [62]), we distinguish three classes of $\mathcal{CP}$-violating phases in $\mathcal{H}_{eff}$:

1. *Flavor-blind phases.* For a given Lagrangian, only the relative phases between coupling constants are relevant. So, it seems natural to identify the phase of $a_1$ in Eq. (2.17) as the phase of $\mathcal{H}_{eff}$ relative to the rest of the Lagrangian, while the relative phases among the coefficients would originate from the (unknown) physics behind the MFV expansion. We call this a flavor-blind phase because through a redefinition of the fields, it is in principle possible to remove the phase of $a_1$ but at the cost of making some non-flavored Lagrangian parameter(s) complex. The induced EDM are typically very large in this case (think of $\theta_{QCD}$ in the SM). So, whenever present, the contribution of this phase to the EDMs dominates. If $\operatorname{Im} a_1$ is $\mathcal{O}(1)$, it is so large that $\Lambda$ has to be far above the TeV scale. A new mechanism, presumably unrelated to MFV, should exist to deal with these flavor-blind $\mathcal{CP}$ violating phases, and ensure $\operatorname{Im} a_1 \ll 1$.

2. *Flavor-diagonal phases.* Because we have written $\mathbf{X}_8$ in terms of hermitian combinations of spurions, its diagonal entries are complex if and only if the coefficients $a_i$ have imaginary parts. We call them flavor diagonal phases. Note that $[\mathbf{X}_u, \mathbf{X}_d]$, $[\mathbf{X}_u, \mathbf{X}_d^2]$, and $[\mathbf{X}_u^2, \mathbf{X}_d]$ have vanishing entries on the diagonal because $\mathbf{X}_{u,d}$ are hermitian, and it is always possible to go in the basis where either $\mathbf{X}_d$ or $\mathbf{X}_u$ is diagonal. Compared to the flavor-blind phase, the contributions of flavor-diagonal phases to the EDM are significantly suppressed for the first two generations, because $\mathbf{X}_8^{11}, \mathbf{X}_8^{22} \ll 1$. Whether this suppression is sufficient depends on the details of the model under consideration, and the hermiticity of the MFV expansions is often required to bring down $\Lambda$ to within reach of the LHC. In that case, $a_i$ are real and flavor-diagonal phases are absent.

3. *Flavor non-diagonal phases.* All the other phases occur only together with some flavor transitions. This includes in particular the CKM phase present in the spurions. Their contributions to

[3]The separation between these two sources is blurry because traces of combinations of spurions can be complex if the spurions involve a $\mathcal{CP}$-violating phase, see Eq. (1.32). Such traces are always understood in the coefficients since they arise from Cayley-Hamilton identities. Worse, they also arise when electroweak radiative corrections to the coefficients are computed [21]. So, as a matter of principle, a strict $\mathcal{CP}$-limit on the MFV coefficients cannot be imposed without forcing the spurions to be also $\mathcal{CP}$-conserving [23].

the EDM does not arise at leading order, but would require dressing the effective operator with some electroweak corrections. A dedicated numerical study is needed in this case to estimate the impact on EDM because it is very dependent on the specific dynamics of the model under consideration. For example, in supersymmetry, such contributions are typically sufficiently suppressed to pass the experimental bounds even for relatively low sparticle masses. As was checked in Ref. [23], this even holds for the leptonic sector, where the spurion phases could be much larger than the CKM phase.

## 2.3 Why should we trust MFV?

The MFV hypothesis severely restricts the NP flavor structures, so much so that FCNC constraints are again compatible with new TeV-scale particles. Even if this is very welcome, both experimentally for the LHC and theoretically for the hierarchy puzzle, one may be left a bit unsatisfied. In all this, there is no clue about the origin of MFV itself. Ideally, we would like to derive it from some full-fledged dynamical mechanism. This is a very ambitious program since its origin may lie in the physics responsible for the observed patterns of quark and lepton masses and mixings. In other words, MFV may be explained only once a comprehensive solution to all the flavor issues is found, and this is still a long way off.

Nevertheless, there have been many attempts in that direction, inspired from previous works on horizontal family symmetries [63], or starting by gauging some flavored symmetry [64]. Examples of studies of the $G_F$ spontaneous breaking can be found in Ref. [65]. In this latter case, a potential hazard would be the presence of light Goldstone bosons, motivating the use of discrete symmetries instead of the continuous $SU(3)$s [66]. Finally, several works have tried to minimally peek beyond MFV by accounting dynamically for the large splitting between the first two and the third generations. The idea is to allow for explicit $G_F$ breaking along the third direction only, so that a large $U(2)^5$ subgroup of $U(3)^5$ is preserved, at least in a first approximation [67].

In the present work, we will stick to a purely phenomenological point of view. Indeed, explaining the origin or the internal structures of the spurions dynamically is not necessary to interpret the MFV hypothesis in very meaningful and universal ways, as we now briefly discuss.

### 2.3.1 Utilitarian interpretation

As presented in the previous section, MFV is at the very least a convenient tool. First, it offers an improved parametrization for any flavor coupling, independent of the basis chosen for the quark fields. Instead of working with the ambiguous values of the couplings in some fermion basis, one deals with the value of the coefficients of the expansions. There are as many free parameters in both descriptions. In this sense, constructing such expansions should be a necessary step in the phenomenological analysis of any NP model. Second, the numerical size of the coefficients in these expansions is the only meaningful measure of the naturality of the new flavor structures. After all, it would not be consistent to say that a NP flavor coupling is unnatural if it is no more fine-tuned than those of the SM. So, MFV could be viewed as an **improved dimensional analysis** tool, designed to tackle the highly hierarchical flavor sector.

### 2.3.2 Pragmatic interpretation

There is another similar though slightly more physically-oriented perspective on the MFV hypothesis. Let us assume that some NP exists, whose dynamics is blind to the flavor of the fields. In practice, this means that its flavor sector is trivial, and the only $G_F$-breaking term in the whole SM plus NP Lagrangian are the usual Yukawa couplings only.

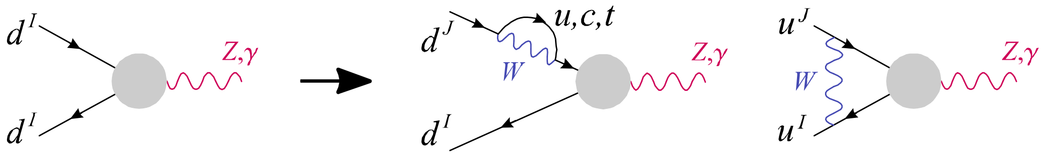


Figure 2.3: Electroweak radiative corrections inducing flavor-violating operators out of a flavor-blind NP couplings $Z_\mu \bar{d}^I \gamma^\mu d^I$.

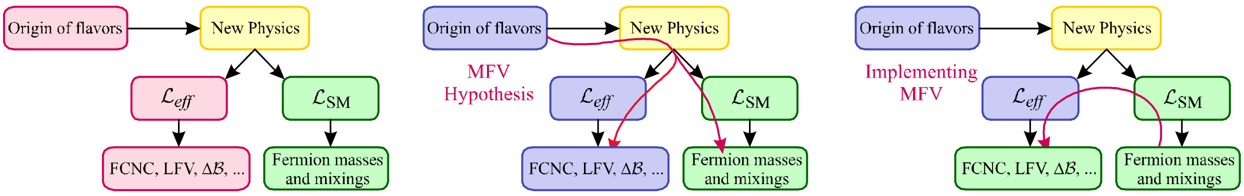


Figure 2.4: Schematic representation of the redundancy interpretation of the MFV hypothesis. When both the SM and the NP flavor structures derive from a minimal set of fundamental spurions, the latter can be parametrized in terms of the former, thereby inheriting their hierarchies and solving the flavor puzzles.

Though this looks promising to control the NP impact on the FCNC, such a flavor-blind NP setting is not tenable because the SM is not flavor blind. The non-trivial SM flavor mixings will spill onto the NP flavor sector through radiative corrections, see Fig. 2.3. At least at the loop level, the flavor-blind NP dynamics combined with the SM flavor mixings will generate new contributions to the FCNC. This is where MFV enters in the picture since all the flavor transitions remain tuned by the Yukawa couplings, as they were in the SM. Actually, the discussion made in the first chapter now applies to the SM plus NP dynamics. MFV is not a hypothesis in this case; it must be strictly valid whenever the only $G_F$ breaking structures are the Yukawa couplings.

Besides the radiative corrections of Fig. 2.3, the spill over of the SM flavor mixing on the NP dynamics also come from renormalization group evolution. For instance, the dynamics at the origin of the flavor structure could ensure that the NP flavor sector is flavor blind, but only at a certain energy scale. When run down to the TeV scale, flavor-blindness would be lost. Again in this case, MFV is strictly valid since the evolution will be tuned by the Yukawa couplings.

In conclusion, MFV emerges as **the least acceptable flavor violation** for the NP sector. We cannot assume it is blinder to flavor because of the ever-present SM flavor dynamics. Said differently, the MFV pattern of flavor mixings are those one should at least allow to ensure stability under electroweak radiative corrections.

### 2.3.3 Redundancy interpretation

MFV can be understood as statement about the mechanism at the origin of the flavor structures. The basic idea is that whatever the physics at the high-scale, it is always possible to promote its fundamental flavor structures to spurions to make it (artificially) flavor symmetric. Thus, what MFV asks is for these spurions to be necessary and sufficient to generate the Yukawa couplings $\mathbf{Y}_{u,d,e}$. In that case, we actually do not need the precise forms of the NP spurions. They can be traded for $\mathbf{Y}_{u,d,e}$, in terms of which all the low-scale NP flavor couplings can then be reconstructed, see Fig. 2.4.

Let us make this statement a bit more explicit. Imagine a low-energy theory with two elementary flavor couplings **A** and **B**. At the very high scale, some flavor dynamics is active and introduces a single explicit breaking of $G_F$, which we call **X**. The two low-energy flavor couplings are induced by this elementary flavor breaking, so it must be possible to express them as:

$$\mathbf{A} = x_1^a \mathbf{1} + x_2^a \mathbf{X} + x_3^a \mathbf{X}^2 \ , \ \mathbf{B} = x_1^b \mathbf{1} + x_2^b \mathbf{X} + x_3^b \mathbf{X}^2 \ . \tag{2.19}$$

A priori, if the dynamics at the origin of the flavor structures was known, the various coefficients $x_{1,2,3}^{a,b}$ could be computed explicitly. Lacking this, we simply assume they are natural, $x_i^{a,b} \lesssim 1$. Also, for these expansions to make sense, powers of **X** must not grow unchecked. A sufficient condition is $\langle \mathbf{X} \rangle \lesssim 1$, since then all $\mathbf{X}^{n>2}$ can be eliminated in terms of $\mathbf{1}$, $\mathbf{X}$, and $\mathbf{X}^2$ while keeping $x_i^{a,b} \lesssim 1$ by using the Cayley-Hamilton identity. Under this condition, from Eq. (2.19), we immediately derive

$$\mathbf{A} = a_1 \mathbf{1} + a_2 \mathbf{B} + a_3 \mathbf{B}^2 \ , \ \ \mathbf{B} = b_1 \mathbf{1} + b_2 \mathbf{A} + b_3 \mathbf{A}^2 \ , \tag{2.20}$$

for some $a_i, b_i$ coefficients. Naturality is preserved since $a_i, b_i \sim \mathcal{O}(1)$ when $x_i^{a,b} \sim \mathcal{O}(1)$, in which case $\langle \mathbf{A} \rangle, \langle \mathbf{B} \rangle \lesssim 1$ when $\langle \mathbf{X} \rangle \lesssim 1$.

This can be readily generalized to more complicated flavor contents. If there are $n$ fundamental spurions, and $m$ flavor couplings, then under the most generic implementation of the MFV hypothesis, all the $m$ flavor couplings are expressed as natural expansions in the $n$ unknown fundamental spurions. In practice, it is possible to eliminate those unknown spurions in favor of the low-energy flavor couplings while preserving the naturality of the expansions. Specifically, we can reexpress $m - n$ flavor couplings as expansions in the remaining $n$ flavor couplings. In this way, one recovers the usual MFV parametrizations in which some couplings (typically the Yukawa couplings) seem to play a more fundamental role than others.

Though simple, this observation is crucial if one wants to probe the physics at the origin of MFV, and more generally the origin of the flavor structures. The fundamental spurions provided by those models need not be some of the low-energy flavor couplings. Evidently, without knowing this fundamental model, only the derived relations among the low-energy flavor couplings are observables, and there is no way to fully reconstruct the fundamental spurions. In other words, the MFV expansions should be understood as the only low-energy observable consequences of the **intrinsic redundancy of the flavor structures**.

Finally, let us make a parallel with the effective description of QCD at low energy. First, remember that if the $u, d, s$ quarks were massless, QCD would have a $SU(3)_L \otimes SU(3)_R$ chiral symmetry. Since mesons and baryons of definite parity do not have degenerate partners with opposite parity, this symmetry is spontaneously broken down to $SU(3)_{V=L+R}$. The dynamics of the corresponding eight Goldstone bosons $\pi, \eta, K$ is then essentially fixed by this symmetry breaking pattern [68].

When quarks are massive, however, the chiral symmetry is explicitly broken. Naively, the whole idea of using the $SU(3)_L \otimes SU(3)_R \rightarrow SU(3)_{V=L+R}$ symmetry breaking pattern may thus seem dubious since the starting symmetry is never exact. However, a crucial piece of information is the uniqueness of the breaking term. There is only one spurion corresponding to the quark masses, aligned with $\text{diag}(m_u, m_d, m_s)$. In practice, it thus suffices to include in the effective theory the perturbations which can be constructed out of this unique spurion, and nothing else. As a corollary, all those perturbations end up related among themselves. This is precisely the same idea with MFV: all the flavor structures are related because they are assumed to all derive from a limited number of fundamental breaking terms.

This analogy also illustrates that trying to reconstruct the fundamental theory at the origin of the flavor structures out of MFV could well be illusory. The low-energy meson dynamics cares only about the chiral symmetry properties of the quark mass term, not its dynamical origin. We now know that quark masses arise from the spontaneous breaking of an unrelated gauge symmetry, that is, from the Higgs mechanism in the electroweak sector. Clearly, this could never have been guessed looking only at the strong interactions among mesons.

## 2.4 Application to the Minimal Supersymmetric Standard Model

The Minimal Supersymmetric extension of the SM (MSSM) particle content is obtained from that of the SM by promoting each field to a superfield, i.e., by associating superpartners to all of them. The gauge group is the same as in the SM, and superfield have definite gauge quantum numbers (for an introductory review, see e.g. Ref. [10]). At this level, supersymmetrizing the SM is very neat and does not introduce new free parameters. The only difference with the SM is that two Higgs doublets are needed to give mass to all the matter fermions, because the Yukawa couplings must derive from a holomorphic superpotential[4]

$$\mathcal{W}_{\text{Yukawa}} = -U^I \mathbf{Y}_u^{IJ} Q^J H_u + D^I \mathbf{Y}_d^{IJ} Q^J H_d + E^I \mathbf{Y}_e^{IJ} L^J H_d - \mu H_u H_d \;, \tag{2.21}$$

where $I, J, K = 1, 2, 3$ denote flavor indices. With $\mathbf{m}_u = v_u \mathbf{Y}_u$ and $\mathbf{m}_d = v_d \mathbf{Y}_d$ with $v_{u,d} = \langle 0|H_{u,d}|0\rangle$, the Yukawa couplings now have similar values when $\tan\beta \equiv v_u/v_d$ becomes large, and their largest entries even unify when $\tan\beta \approx m_t/m_b \approx 50$.

The main difficulty with supersymmetry is actually to break it. This is necessary to lift the mass degeneracy among superpartners, and to allow for a successful electroweak SSB. However, the precise mechanism at its origin is not established yet. In the MSSM, an effective approach is adopted to deal with this uncertainty. All the possible gauge-invariant breaking terms are added by hand, with the constraint that they should not spoil the nice renormalization features of the MSSM. Such a soft breaking of supersymmetry allows for dimensionful breaking terms only. Specifically, concentrating on those breaking terms involving flavored fields, gauge invariance permits to introduce

$$\begin{aligned}\mathcal{L}_{\text{Soft}} = &-\tilde{Q}^{\dagger I}(\mathbf{m}_Q^2)^{IJ}\tilde{Q}^J - \tilde{U}^I(\mathbf{m}_U^2)^{IJ}\tilde{U}^{\dagger J} - \tilde{D}^I(\mathbf{m}_D^2)^{IJ}\tilde{D}^{\dagger J} - \tilde{L}^\dagger(\mathbf{m}_L^2)^{IJ}\tilde{L} - \tilde{E}(\mathbf{m}_E^2)^{IJ}\tilde{E}^\dagger \\ &- \tilde{U}^I \mathbf{A}_u^{IJ}\tilde{Q}^J H_u + \tilde{D}^I \mathbf{A}_d^{IJ}\tilde{Q}^J H_d + \tilde{E}^I \mathbf{A}_e^{IJ}\tilde{L}^J H_d + h.c. \;,\end{aligned} \tag{2.22}$$

where tilde indicate the scalar squark and slepton fields. With $\mathbf{m}^2_{Q,U,D,L,E}$ arbitrary hermitian scalar mass terms, and $\mathbf{A}^{IJ}_{u,d,e}$ generic complex matrices inducing left-right sfermion mixings after SSB, this sector introduces a plethora of new free parameters. This is clearly not very satisfying. Worse, these new couplings a priori induce new contributions to the tightly constrained FCNC. If Sec. 2.1 is any guide, these couplings should end up extremely fine-tuned when sparticles have masses around the TeV scale.

### 2.4.1 Supersymmetric flavor puzzle

Let us briefly check this statement, using again the language of the flavor symmetry and its breaking. The MSSM gauge sector is invariant under the same flavor group as the SM, i.e. $G_F = U(3)^5$, provided $G_F$ now acts simultaneously on flavored fermions and their superpartners, i.e.

$$Q \to g_Q Q \Leftrightarrow (\tilde{u}_L^I, \tilde{d}_L^I, u_L^I, d_L^I) \to g_Q^{IJ}(\tilde{u}_L^J, \tilde{d}_L^J, u_L^J, d_L^J) \;,\; U \to U g_U \Leftrightarrow (\tilde{u}_R^{I\dagger}, \bar{u}_R^I) \to (\tilde{u}_R^{J\dagger}, \bar{u}_R^J) g_Q^{JI} \;, \tag{2.23}$$

and similarly for the other matter supermultiplets $D$, $L$ and $E$. The main difference with the SM is, however, the presence of many new flavor structures. If one insists on maintaining $G_F$ as an exact symmetry for the whole MSSM, besides the Yukawa couplings, the soft-breaking terms should also be included into the list of spurions. They can thus be used to construct $G_F$-invariant FCNC operator.

Specifically, consider the $Z$ penguin, Eq. (1.17). With many non-diagonal spurions at our disposal, there are now many ways to construct a flavor-violating operator, for example as

$$\begin{aligned}\mathcal{H}_{eff}^{\text{SUSY}} = &\frac{1}{\Lambda^4}(\bar{Q}(\mathbf{m}_Q^2 + \mathbf{A}_u^\dagger \mathbf{A}_u + \mathbf{A}_d^\dagger \mathbf{A}_d + \mathbf{Y}_u^\dagger \mathbf{m}_U^2 \mathbf{Y}_u + ...)\gamma_\mu Q) H^\dagger D^\mu H \\ &+ \frac{1}{\Lambda^4}(D(\mathbf{m}_D^2 + \mathbf{A}_d \mathbf{A}_d^\dagger + \mathbf{Y}_d \mathbf{m}_Q^2 \mathbf{Y}_d^\dagger + ...)\gamma_\mu \bar{D}) H^\dagger D^\mu H + ... \;,\end{aligned} \tag{2.24}$$

[4]Baryon and lepton numbers are assumed conserved here. Their violation will be discussed in Chapter 3.

where $\Lambda$ denotes the supersymmetric scale, and the overall suppression is really in $1/\Lambda^2$ since $\mathbf{m}^2_{Q,U,D} \sim \Lambda^2$ and $\mathbf{A}_{u,d} \sim \Lambda$. Actually, one often defines the dimensionless **mass insertions** as $\delta_Q \equiv \mathbf{m}^2_Q/\Lambda^2$, and similarly for the other couplings. Comparing with the previous model-independent analysis, this supersymmetric scale must be well above the TeV scale if these mass insertions are all of $\mathcal{O}(1)$. Actually, the best way to maintain $\Lambda$ at around the TeV scale is to force e.g.

$$\mathbf{m}^2_Q \sim \Lambda^2 \mathbf{Y}^\dagger_u \mathbf{Y}_u \ \rightarrow \delta_Q \sim V^*_{tI} V_{tJ} \ . \tag{2.25}$$

Mass insertions are then tuned by the same CKM factor as for the SM contributions. Similarly, if we can write $\mathbf{m}^2_D \sim \Lambda^2 \mathbf{Y}_d \mathbf{Y}^\dagger_d$, the SM chiral suppression is recovered for the second operator, see Eq. (1.26).

This observation can be repeated for the other couplings, showing that supersymmetry in general suffers from the NP flavor puzzle [18]. Either its scale is very high, or its many flavor couplings have to be fine-tuned.

### 2.4.2 MFV expansions for soft-breaking terms

Applying the first precept of MFV, all the soft-breaking terms should be expressed as polynomial expansions in the Yukawa couplings. Specifically, from Eq. (2.17), we write [21, 69]

$$\begin{aligned}\mathbf{m}^2_Q/m^2_0 &= a^q_1 \mathbf{1} + a^q_2 \mathbf{X}_u + a^q_3 \mathbf{X}_d + a^q_4 \mathbf{X}^2_u + a^q_5 \mathbf{X}^2_d + a^q_6 \{\mathbf{X}_u, \mathbf{X}_d\} \\ &\quad + b^q_1 \, i[\mathbf{X}_u, \mathbf{X}_d] + b^q_2 \, i[\mathbf{X}^2_u, \mathbf{X}_d] + b^q_3 \, i[\mathbf{X}_u, \mathbf{X}^2_d] \ ,\end{aligned} \tag{2.26a}$$

$$\begin{aligned}\mathbf{m}^2_{U,D}/m^2_0 &= a^{u,d}_1 \mathbf{1} + \mathbf{Y}_{u,d} \, (a^{u,d}_2 \mathbf{1} + a^{u,d}_3 \mathbf{X}_u + a^{u,d}_4 \mathbf{X}_d + a^{u,d}_5 \mathbf{X}^2_{d,u} + a^q_6 \{\mathbf{X}_u, \mathbf{X}_d\}) \, \mathbf{Y}^\dagger_{u,d} \\ &\quad + \mathbf{Y}_{u,d} \, (b^{u,d}_1 \, i[\mathbf{X}_u, \mathbf{X}_d] + b^{u,d}_2 \, i[\mathbf{X}^2_u, \mathbf{X}_d] + b^{u,d}_3 \, i[\mathbf{X}_u, \mathbf{X}^2_d]) \, \mathbf{Y}^\dagger_{u,d} \ ,\end{aligned} \tag{2.26b}$$

$$\begin{aligned}\mathbf{A}_{u,d}/A_0 &= \mathbf{Y}_{u,d} \, (c^{u,d}_1 \mathbf{1} + c^{u,d}_2 \mathbf{X}_u + c^{u,d}_3 \mathbf{X}_d + c^{u,d}_4 \mathbf{X}^2_u + c^{u,d}_5 \mathbf{X}^2_d + c^{u,d}_6 \{\mathbf{X}_u, \mathbf{X}_d\} \\ &\quad + c^{u,d}_7 \, i[\mathbf{X}_u, \mathbf{X}_d] + c^{u,d}_8 \, i[\mathbf{X}^2_u, \mathbf{X}_d] + c^{u,d}_9 \, i[\mathbf{X}_u, \mathbf{X}^2_d]) \ ,\end{aligned} \tag{2.26c}$$

where $\mathbf{X}_{u,d} \equiv \mathbf{Y}^\dagger_{u,d} \mathbf{Y}_{u,d}$, and both $A_0$ and $m_0$ set the overall mass scale. As before, as long as the coefficients are left free, this is a fully generic parametrization. Its main interest is that the real coefficients $a^{q,u,d}_i$ and $b^{q,u,d}_i$ and the complex coefficients $c^{u,d}_i$ are independent of the flavor basis in which the (s)quark fields are defined, so they offer an unambiguous measure of the flavor violation present in the soft-breaking terms. To impose MFV, it now suffices to allow for $\mathcal{O}(1)$ coefficients only. Then, clearly, the MSSM flavor puzzles are alleviated since all the mass insertions are tuned by the CKM mixings.

The spurion expansion, with or without the MFV constraint on the coefficients, has many other desirable features:

1. The soft terms at any energy scale $Q$ admit expansions of the form (2.26), where both soft terms and Yukawa couplings are understood as those at the scale $Q$. Thus, the running of the soft masses and trilinear terms can be represented by that of the flavor coefficients. Their renormalization group equations (RGEs) were studied in Refs. [21, 70]. Typically, not only are the evolutions of the coefficients $a_{i\neq 1}[Q], b_i[Q], c_{i\neq 1}[Q]$ from $Q = M_{\text{GUT}}$ down to the TeV scale smooth and bounded, they even exhibit infrared quasi-fixed points ("quasi" because their values depend on those of the other MSSM parameters).

2. The $\beta$-functions of the soft masses and trilinear terms are naturally compatible with the expansions (2.26), and the running of the various coefficients sum up different physical effects. For example, the leading coefficients $a^{q,u,d}_1[Q], c^{u,d}_1[Q]$ entirely encode the dominant flavor-blind evolution, while subleading terms evolve separately.

3. The phenomenological impact of the flavor mixing induced by the off-diagonal soft-term entries can immediately be assessed since the MFV limit is recovered when all the coefficients are $\mathcal{O}(1)$. This means that one can directly spot potentially dangerous sources of new FCNCs simply by looking at the relative sizes of the coefficients at the low-energy scale.

4. Starting with flavor-blind soft-breaking terms, $a_i^{q,u,d}[M_{\rm GUT}] = c_i^{u,d}[M_{\rm GUT}] = \delta_{i1}$, $b_i^{q,u,d}[M_{\rm GUT}] = 0$, as in the constrained MSSM, the coefficients at the low scale are all MFV-like, i.e., $\mathcal{O}(1)$ or smaller [71]. More generally, the logarithmic running with small coupling constants cannot upset initial MFV-like boundary conditions at the GUT scale. The converse is not true though, because of the presence of the aforementioned quasi-fixed points [21].

5. The new $\mathcal{CP}$-violating phases $b_i^{q,u,d}[M_{\rm GUT}]$ and $\operatorname{Im} c_i^{u,d}[M_{\rm GUT}]$ are immediately factored out from the $\mathcal{CP}$-violating effects induced by the CKM phase present in the spurions $\mathbf{Y}_{u,d}$. Note that if $b_i^{q,u,d}[M_{\rm GUT}] = 0$ and $\operatorname{Im} c_i^{u,d}[M_{\rm GUT}] = 0$, their values at the electroweak scale are entirely induced by the $\mathcal{CP}$-violating phase of $V_{CKM}$, and end up tiny. In this respect, the $\mathcal{CP}$-violating phases of $c_1^u[Q]$ and $c_1^d[Q]$ are a bit special. Being flavor blind, they should be considered along with those of the other complex parameters of the MSSM such as $\mu$ and the gaugino masses [23]. All those flavor-blind phases need to be tiny to pass EDM constraints with TeV-scale sparticles.

6. In the general case, the spurion expansions do not reduce the number of free parameters: there are as many coefficients as there are entries in the three-by-three soft-breaking terms. The situation changes when MFV is imposed, because then at most five terms are needed for each expansion (see Eq. (2.16)). Further, for phenomenological analyses, it is even often sufficient to keep just $a_{1,2}^{q,u,d}$ and $c_1^{u,d}$ to account for realistic flavor mixing and mass patterns in the squark sector, especially when $\tan\beta$ is not large and $\mathbf{Y}_d \ll \mathbf{Y}_u$.

MFV restricts not only the size of the flavor mixing in the squark sector, it also restricts the squark mass spectrum tuned by the eigenvalues of the soft terms. When all the coefficients are $\mathcal{O}(1)$, the expansions (2.26) are dominated by their leading term. There are then three groups of nearly degenerate squarks: the six left-type squarks $\tilde{d}_L$, $\tilde{s}_L$, $\tilde{b}_L$, $\tilde{u}_L$, $\tilde{c}_L$, $\tilde{t}_L$, the three right-type up-squarks $\tilde{u}_R$, $\tilde{c}_R$, $\tilde{t}_R$, and the three right-type down-squarks $\tilde{d}_R$, $\tilde{s}_R$, $\tilde{b}_R$.

This said, MFV permits also to easily and straightforwardly parametrize Natural SUSY [72] boundary conditions, where the third-generation squarks are split from the first two [73]. Consider for example the expansion $\mathbf{m}_Q^2[M_{\rm GUT}] = m_0^2(\mathbf{1} - \alpha_q \mathbf{Y}_u^\dagger \mathbf{Y}_u \langle \mathbf{Y}_u^\dagger \mathbf{Y}_u \rangle^{-1})$. Provided the free parameter $\alpha_q$ is close to one, $\mathbf{m}_Q^2$ has its first two eigenvalues nearly degenerate and much larger than the third (as can be trivially seen in the basis where $\mathbf{Y}_u$ is diagonal). At the low scale $\tilde{t}_L$ and $\tilde{b}_L$ then end up much lighter than all the other squarks. But, even if left squark masses are highly hierarchical, this boundary condition respects MFV since $\langle \mathbf{Y}_u^\dagger \mathbf{Y}_u \rangle \approx y_t^2$ is of $\mathcal{O}(1)$ at all scales. So, once evolved to the low scale, we can immediately predict that these initial conditions should be compatible with flavor constraints. Other scenarios can be constructed along the same lines. For instance, to also split the $\tilde{t}_R$ from the first- and second-generation squarks, one can further impose $\mathbf{m}_U^2 = m_0^2(\mathbf{1} - \alpha_u \mathbf{Y}_u \mathbf{Y}_u^\dagger \langle \mathbf{Y}_u \mathbf{Y}_u^\dagger \rangle^{-1})$ which is also compatible with the MFV principle when $\alpha_u \approx 1$. On the other hand, there is no way to split the right sbottom from the first two generations without moving away from MFV. Indeed, all the non-trivial terms in the expansion of $\mathbf{m}_D^2$ are sandwiched between $\mathbf{Y}_d$ and $\mathbf{Y}_d^\dagger$, which are small when $\tan\beta$ is not very large.

# Chapter 3

# Baryon and lepton numbers

Since the dawn of particle physics, the apparent stability of the proton has been puzzling. On one hand, its perfect stability was accepted simply from the day-to-day observation that all the ordinary matter around us does not disappear. But on the other, this stability never emerged naturally. It did not derive from an obvious symmetry, a basic principle, or a dynamical requirement. Instead, it had to be imposed by hand.

All started in the late 1920s, at the birth of the Dirac theory. The proton was initially thought by Dirac and Weyl to be the "negative energy" partner of the electron. But Weyl quickly realized that this would open the annihilation channel $p + e \to \gamma$, endangering the stability of all known atoms. To prevent this process, in analogy with the conserved electric charge, Weyl introduced a new conserved charge, carried exclusively by the proton [74].

A similar story happened about ten years later. By the end of the 30s, the neutron and positron had been discovered, along with beta decay processes like $n^0 \to p^+ e^- \nu$. Yet, no indication that $p^+ \not\to e^+ \gamma$ or of the apparently innocuous $n^0 \not\to p^- e^+ \nu$. To explain this, Stuckelberg [75] introduced again an additive conserved charge which he dubbed the "heavy charge" carried by strong interacting particles. Neutrons and protons were given an equal charge, their antiparticles had the opposite charge, while the photon and leptons were neutral. In this respect, it should be noted that a conserved lepton number would also prevent proton decay (the spin 1/2 proton must decay into an odd number of fermions, and only leptons are lighter than the proton). However, by the end of the 30s, Majorana had already proposed his theory [16] and a conserved lepton number was not seen as natural or promising. The situation took a long time to clear up in the lepton sector essentially because of the difficulty to directly observe neutrinos. It is only in the early sixties that the individual lepton flavor numbers were observed to be conserved, thereby explaining why $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ but $\mu \not\to e\gamma$.

Another jump of ten years and the story repeats itself. This time, the pions had been discovered, along with an additional nucleon-like fermionic state, the $\Lambda$. To explain why e.g. $\pi^+ \to e^+ \nu$ and $\Lambda^0 \to p^+ \pi^-$ but $p^+ \not\to e^+ \pi^0$ and $\Lambda^0 \not\to p^- \pi^+$, Wigner redefined Stuckelberg's heavy charge to exclude mesons [76]. Finally, the modern definition of baryon number ($\mathcal{B}$) was born: proton, neutron, $\Lambda$,... have $\mathcal{B} = 1$, their antiparticle $\mathcal{B} = -1$, and all the other particles are neutral. This difference between mesons and baryons was clarified once quarks were introduced in the 60s [77]. With quarks assigned $\mathcal{B} = +1/3$ and antiquarks $\mathcal{B} = -1/3$, the $q\bar{q}$ mesons have $\mathcal{B} = 0$ but the $qqq$ baryons have $\mathcal{B} = 1$.

All this was neatly embedded in the SM. Renormalizability combined with color gauge invariance forbids baryon number violation because it would require a coupling with three quarks. Lorentz invariance calls for at least a fourth fermion and the simplest interaction has a dimension greater than four. For instance, if we relax the renormalizability constraint, Weinberg pointed out that the

SM gauge symmetries allow for $\mathcal{B}$ and $\mathcal{L}$ violation through [78]

$$\mathcal{H}_{eff}^{\Delta B,\Delta L}=\frac{\mathcal{C}_0 LHLH}{\Lambda}+\frac{\mathcal{C}_1 LQQQ+\mathcal{C}_2 LQD^\dagger U^\dagger+\mathcal{C}_3 E^\dagger U^\dagger U^\dagger D^\dagger+\mathcal{C}_4 E^\dagger U^\dagger QQ}{\Lambda^2}+...\;, \tag{3.1}$$

where the required $SU(2)_L$ contractions as well as flavor indices are understood, $\Lambda$ is a typical mass scale and the $\mathcal{C}_i$ are dimensionless numerical coefficients. The unique dimension-five operator violates lepton number ($\mathcal{L}$) by two units. Once the Higgs boson gets its vacuum expectation value, it corresponds to a Majorana mass term for the left-handed neutrinos. The dimension-six operators violate both $\mathcal{B}$ and $\mathcal{L}$ by one unit and can induce the decay of the proton.

These terms represent the leading $\mathcal{B}$ and/or $\mathcal{L}$ violating departures from the SM. Experimentally though, they were already tightly constrained in the sixties from the searches for neutrinoless double beta decay and proton decay. In both cases, tremendous sensitivities were achievable because these processes can be tested using macroscopic quantities of readily available materials. This is especially true for proton decay. As Goldhaber put it [79], a simple back-of-the-envelope calculation shows that the proton lifetime has to be greater than about $10^{16}$ years, otherwise the radiations from our own body would quickly kill us. His dedicated experiment in 1954 push this bounds to $10^{21}$ years [80], while current best bounds now stand above $10^{33}$ years for some decay modes, and translate as

$$\Gamma_{p^+}\approx\frac{\alpha_p^2 m_p}{4\pi F_\pi^2}\frac{|\mathcal{C}_i|^2}{\Lambda^4}\lesssim 10^{-62}\text{ GeV }\Rightarrow\begin{cases}\Lambda\gtrsim 10^{11}\text{ TeV} & \text{if } \mathcal{C}_i\sim\mathcal{O}(1)\;,\\ \Lambda\gtrsim 10^{5}\text{ TeV} & \text{if } \mathcal{C}_i\sim\mathcal{O}(m_\nu/v)\;,\\ \Lambda\lesssim 1\text{ TeV} & \text{if } \mathcal{C}_i\sim\mathcal{O}(10^{-22})\;,\end{cases} \tag{3.2}$$

with $\alpha_p\approx 0.0118\,(21)$ GeV$^3$ a non-perturbative constant for the $p^+$ matrix element.

By the end of the sixties, the conservation of both $\mathcal{B}$ and $\mathcal{L}$ were thus generally accepted, though for rather different reasons. The conservation of $\mathcal{B}$ was perceived as a theoretical dogma, obviously confirmed experimentally. That of $\mathcal{L}$, along with the absence of lepton flavor mixing, was seen as a theoretical oddity, more or less acceptable as long as neutrinos are perfectly massless. But in the following decade, everything changed. In the words of Weinberg [78]:

> *Of the supposedly exact conservation laws of physics, two are especially questionable: the conservation of $\mathcal{B}$ and $\mathcal{L}$. As far as we know, there is no necessity for an a priori principle of $\mathcal{B}$ and $\mathcal{L}$ conservation (...) In contrast with the conservation of charge, color, and energy and momentum, the conservation of $\mathcal{B}$ and $\mathcal{L}$ are almost certainly not unbroken local symmetry.*

There are essentially four theoretical reasons behind this dramatic shift in attitude in the seventies.

1. **Anomalies**. As we will discuss later on in this chapter, even though $\mathcal{B}$ and $\mathcal{L}$ are exact symmetries of the SM Lagrangians, they do not both survive quantization. The $\mathcal{B}+\mathcal{L}$ combination is anomalous, a fact discovered by 't Hooft in 1976 [15]. So, even in the SM, $\mathcal{B}$ and $\mathcal{L}$ are not true symmetries!

2. **Grand Unified Theories** (GUT). The SM is constructed as a spontaneously broken gauge symmetry. This same receipt can be used to derive the SM from a simpler gauge group, broken at a higher energy scale. The first such proposal was the Georgi–Glashow $SU(5)$ model presented in 1974 [8]. It neatly explains the quantization of the hypercharge and is able to predict the Weinberg angle of the SM. But, it also unifies quark and leptons into representations of $SU(5)$, so intrinsically, $\mathcal{B}$ and $\mathcal{L}$ are no longer well defined (though $\mathcal{B}-\mathcal{L}$ happens to survive). Consequently, GUT theories predict proton decay. In the original Georgi-Glashow model, unification occurs somewhere around $10^{14-15}$ GeV, hence setting $\Lambda$ to this value in Eq. (3.1), the typical proton lifetime should be around $10^{30}$ years.

3. **Left-right symmetries**. The SM treats left and right-handed fermions very differently. This could be the result of an asymmetric breakdown of an initially left-right symmetric gauge group. This hypothesis is viable only if the matter content can be organized in a left-right symmetric fashion, which clearly asks for a flavor triplet of right-handed neutrinos. These particles being necessarily neutral under the SM gauge group, nothing prevents a $\Delta\mathcal{L} = 2$ Majorana mass term [16] for them, hence lepton number conservation as well as lepton flavor conservation are all but automatic. It is in this context that the seesaw mechanism was designed in 1977 [1].

4. **Sakharov's conditions**. At the end of the sixties, the Big Bang theory was more or less universally accepted, though many aspects of the evolution of the universe were still rather obscure, in particular the current prevalence of matter over antimatter starting with a perfectly balanced universe. In 1967, Sakharov enumerated three conditions for a baryon asymmetry [6]: there must be $\mathcal{CP}$ violation, $\mathcal{B}$ violation, and an out-of-thermal equilibrium transition. Note that nowadays, the cosmological necessity of $\mathcal{B}$ violation is far less clear. First, Fukugita and Yanagida showed in 1986 that only $\mathcal{L}$ violation would suffice [81], since the SM anomaly can generate a $\mathcal{B}$ asymmetry out of it. Second, a large quantity of dark matter is now known to fill the universe. Models exist where this dark matter is charged under $\mathcal{L}$ and/or $\mathcal{B}$, so that no $\mathcal{B}$ or $\mathcal{L}$ violation is actually needed.

For all these reasons, the conservation of $\mathcal{B}$ was no longer believed to hold, and the proton was expected to decay at some point. So, in the eighties, several dedicated experiments started, aiming at the range of lifetimes predicted by GUT models. No signal were ever found. But, in an amazing twist of Nature, these experiments did not draw blank. They were instrumental in the discovery of neutrino oscillations, thereby disproving the conservation of lepton flavors and casting a big shadow on the conservation of $\mathcal{L}$! Actually, since in any case $\mathcal{B}+\mathcal{L}$ is not a symmetry, hints for the $\Delta\mathcal{L} = 2$ seesaw could even point towards $\Delta\mathcal{B} = 2$ effects like neutron-antineutron oscillations. So, finally, experiment starts to support the idea that neither $\mathcal{B}$ nor $\mathcal{L}$ are exact symmetries of Nature.

Nowadays, the status of $\mathcal{B}$ and $\mathcal{L}$ has not evolved much. Though they are most probably not exact symmetries at high energy, whether at the GUT or seesaw scale, they are still most of the time imposed to be conserved at low energy, at least perturbatively, to prevent too fast proton decay, $0\nu\beta\beta$ transitions, or neutron-antineutron oscillations. There is no consensus on a mechanism ensuring their low-energy near conservation though, and this is usually an ad hoc phenomenological assumption. On the experimental side, searches are still going on for $\mathcal{B}$ and/or $\mathcal{L}$ violating phenomena, though with less enthusiasm and dynamism than in the eighties.

## 3.1 Anomalies and the fate of the $U(1)$s

In Chapter 1, we identified the $G_F = U(3)^5$ flavor symmetry from the invariance of the SM gauge interactions under individual flavor-space rotations of each fermion species. Though exact classically, this symmetry is not preserved under quantization. Because it treats separately the left and right-handed fermion fields, part of it is broken by **chiral anomalies** [82]. Said differently, this global symmetry is not compatible with the introduction of gauge interactions (even though it was identified precisely looking at the gauge part of the SM Lagrangian), and its currents are not conserved.

Specifically, for the combined $G_F \otimes G_{SM}$ symmetry group, the only non-trivial **triangle anomalies** are generically of the form (Fig. 3.1)

$$\partial^\mu \bar{\psi}_L \gamma_\mu T^a_{G_F} \psi_L = -\frac{g^2}{32\pi^2} F^b_{\mu\nu} \tilde{F}^c_{\rho\sigma} \,\mathrm{Tr}(T^a_{G_F}\{T^b_{G_{SM}}, T^c_{G_{SM}}\}) \;, \tag{3.3}$$

where $T^b_{G_{SM}}$ is the SM generator corresponding to the fermion $\psi_L$, $F^a_{\mu\nu}$ and $g$ the corresponding SM field strength and coupling constant ($\tilde{F}^a_{\mu\nu} = 1/2\varepsilon^{\mu\nu\rho\sigma} F^a_{\rho\sigma}$), and all the SM fermions are defined

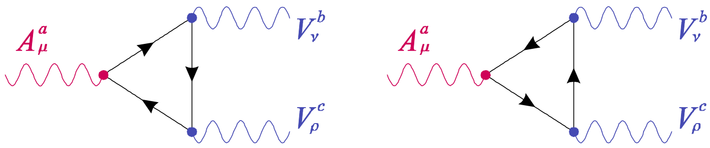


Figure 3.1: Triangle anomalies breaking the conservation of the chiral flavor currents (red) when the intermediate fermions are coupled to gauge fields (blue).

as left-handed fields as in Table 1.1. Note that the anomalies with less than two gauge fields are irrelevant since there are no field strength associated with the global symmetry $G_F$. For example, the $U(3)$ factors of $G_F$ are not compatible with each other since $\mathrm{Tr}(T^a_{G_F}\{T^b_{G_F},T^c_{G_F}\}) \neq 0$, but this has no physical consequence. As is well-known, the triangle anomalies with three gauge fields do not vanish either, but cancel out when summed over the five fermion species.

The flavor $SU(3)$ are not anomalous since $\mathrm{Tr}(T^a_{SU(3)_\psi}\{T^b_{GSM},T^c_{GSM}\}) = \mathrm{Tr}\,T^a_{SU(3)_\psi}\,\mathrm{Tr}\{T^b_{GSM},T^c_{GSM}\}$ and $SU(3)$ generators are traceless. So, the $G_F$ symmetry is only plagued by the **singlet anomalies**, those with $T_{U(1)_\psi} \sim \mathbf{1}$, since the Casimir invariants $\mathrm{Tr}\{T^b_{GSM},T^c_{GSM}\}$ do not vanish. None of the five $U(1)$ symmetries survives quantization, and their associated currents catch the anomalous terms

$$\partial^\mu \bar{\psi}_L \gamma_\mu \psi_L = -\frac{N_f}{16\pi^2}\left[d_L C_C g_s^2 G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} + d_C C_L g^2 W^i_{\mu\nu}\tilde{W}^{i,\mu\nu} + d_C d_L C_Y g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu}\right] . \tag{3.4}$$

The Casimir invariants $C_{C,L,Y}$ are associated to the $SU(3)_C$, $SU(2)_L$, $U(1)_Y$ representations carried by $\psi_L$, and $d_{C,L}$ are their corresponding dimensions. The Casimir invariants are normalized such that $C_{C(L)} = 1/2$ for the fundamental $SU(3)_C$ ($SU(2)_L$) representation, and $C_Y = Y^2/4$. In details,

$$\begin{pmatrix} \partial_\mu J^\mu_Q \\ \partial_\mu J^\mu_U \\ \partial_\mu J^\mu_D \\ \partial_\mu J^\mu_L \\ \partial_\mu J^\mu_E \end{pmatrix} = -\frac{N_f}{16\pi^2}\begin{pmatrix} 1 & 3/2 & 1/6 \\ 1/2 & 0 & 4/3 \\ 1/2 & 0 & 1/3 \\ 0 & 1/2 & 1/2 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} g_s^2 G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} \\ g^2 W^i_{\mu\nu}\tilde{W}^{i,\mu\nu} \\ g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu} \end{pmatrix} . \tag{3.5}$$

Given the gauge quantum numbers of the SM fermion fields, not all five of these $U(1)$ current anomalies are independent. Actually, since there are only three gauge groups, two anomaly-free combinations must exist. As a first step, let us define

$$\begin{aligned} G_F &= SU(3)^5 \otimes U(1)_Q \otimes U(1)_U \otimes U(1)_D \otimes U(1)_L \otimes U(1)_E \\ &= SU(3)^5 \otimes U(1)_Y \otimes U(1)_{\mathcal{B}} \otimes U(1)_{\mathcal{L}} \otimes U(1)_{\mathcal{PQ}} \otimes U(1)_E , \end{aligned} \tag{3.6}$$

corresponding to the reordering of the five $U(1)$ charges as

$$\begin{pmatrix} J^\mu_Y \\ J^\mu_{\mathcal{B}} \\ J^\mu_{\mathcal{L}} \\ J^\mu_{\mathcal{PQ}} \\ J^\mu_E \end{pmatrix} = \begin{pmatrix} 1/3 & -4/3 & 2/3 & -1 & 2 \\ 1/3 & -1/3 & -1/3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} J^\mu_Q \\ J^\mu_U \\ J^\mu_D \\ J^\mu_L \\ J^\mu_E \end{pmatrix} . \tag{3.7}$$

Combining Eq. (3.5) and (3.7), these currents now have the anomalies:

$$\begin{pmatrix} \partial_\mu J^\mu_Y \\ \partial_\mu J^\mu_{\mathcal{B}} \\ \partial_\mu J^\mu_{\mathcal{L}} \\ \partial_\mu J^\mu_{\mathcal{PQ}} \\ \partial_\mu J^\mu_E \end{pmatrix} = -\frac{N_f}{16\pi^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & 1/2 & -1/2 \\ 1 & 0 & 8/3 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} g_s^2 G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} \\ g^2 W^i_{\mu\nu}\tilde{W}^{i,\mu\nu} \\ g'^2 B_{\mu\nu}\tilde{B}^{\mu\nu} \end{pmatrix} . \tag{3.8}$$

Without surprise, $\partial_\mu J_Y^\mu = 0$ since the fermion charges under this flavored $U(1)$ are precisely aligned with those of the hypercharge $U(1)_Y$, which has to be anomaly-free to be gauged consistently (note though that those two groups are not identified since the Higgs multiplet is charged only under the gauged $U(1)_Y$).

The other anomaly-free combination is that associated with baryon number minus lepton number, $\partial_\mu J_{\mathcal{B}}^\mu - \partial_\mu J_{\mathcal{L}}^\mu = 0$. The corresponding $U(1)_{\mathcal{B}-\mathcal{L}}$ symmetry is the only local $U(1)$ which could in principle be added to the SM. Note though that if gauged, the flavor currents can have a new anomaly $\partial^\mu \bar{\psi}_L \gamma_\mu \psi_L \sim d_C d_L C_{\mathcal{B}-\mathcal{L}} g_{\mathcal{B}-\mathcal{L}}^2 X_{\mu\nu}\tilde{X}^{\mu\nu}$ where $g_{\mathcal{B}-\mathcal{L}}$ and $X^{\mu\nu}$ are the $U(1)_{\mathcal{B}-\mathcal{L}}$ coupling constant and field strength, and $C_{\mathcal{B}-\mathcal{L}} = 2/3,\ 1/3,\ 1/3,\ 2,\ 1$ for $Q$, $U$, $D$, $L$, $E$. As a result, $\partial_\mu J_{\mathcal{B}-\mathcal{L}}^\mu \neq 0$ and the gauge symmetry does not survive quantization. On the other hand, if a right-handed neutrino $N$ is added, its current $\partial^\mu N \gamma_\mu N$ will only have a $X_{\mu\nu}\tilde{X}^{\mu\nu}$ term since $N$ is neutral under the SM gauge groups, and this term precisely cancel that of the other fields so $\partial_\mu J_{\mathcal{B}-\mathcal{L}}^\mu = 0$. This is the situation in many Grand Unified extensions of the SM. For instance, the $SO(10)$ GUT is automatically anomaly free ($\mathrm{Tr}(T^a\{T^b, T^c\}) = 0$ for all representations) and embeds the $U(1)_{\mathcal{B}-\mathcal{L}}$ symmetry among its generators. This is possible because for the fermions to fit in the fundamental 16 representation, a field with precisely the quantum numbers of the right-handed neutrino has to be present.

### 3.1.1 Strong but no weak $\mathcal{CP}$ puzzle

In principle, the SM Lagrangian should be supplemented with three $\mathcal{CP}$-violating interactions among gauge bosons:

$$\mathcal{L}_{\mathcal{C}\not{\mathcal{P}}} = \theta_C \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} + \theta_L \frac{g^2}{16\pi^2} W_{\mu\nu}^i \tilde{W}^{i,\mu\nu} + \theta_Y \frac{g'^2}{16\pi^2} B_{\mu\nu}\tilde{B}^{\mu\nu} \ . \tag{3.9}$$

These interactions can be written as total derivatives, for example $G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} = \partial_\mu[\varepsilon^{\mu\nu\rho\sigma} A_\nu^a (2G_{\rho\sigma}^a - 1/3 g_s f^{abc} A_\rho^b A_\sigma^c)]$ with $f^{abc}$ the $SU(3)_C$ structure constant. Applying Gauss theorem, they do not contribute at the perturbative level (the Feynman rule vanishes). However, for non-abelian groups, there exists non-perturbative configurations of the gauge fields such that the surface term does not vanish, and these interactions could in principle induce physical effects. This is particularly worrying for the first term, which could give rise to sizable $\mathcal{CP}$ violating effects at low-energy, where the strong interaction is dominated by non-perturbative dynamics.

Let us first show that if all the fermions are massless, these interactions can be rotated away thanks to the anomalies. Denoting the number of flavors by $N_f$, and since $U(N_f)_X \sim SU(N_f)_X \otimes U(1)_X$, a flavor transformation can be written as $U_X = \exp(i\alpha_X T^0)\exp(i\alpha_X^a T^a)$ where $T^a$, $a = 1, ..., N_f^2 - 1$ are the $SU(N_f)_X$ generators and $T^0 = \mathbf{1}$ is the $U(1)_X$ generator. Inverting and using the identity $\det(\exp A) = \exp\langle A\rangle$, the $U(1)$ parameters can be extracted using $\alpha_X N_f = \arg\det U_X$. Performing such $U(1)$ rotations, the Lagrangian of the SM is not invariant since the currents are not conserved. According to Noether's Theorem, its variation is

$$\Delta\mathcal{L} = \sum_\psi \alpha_\psi \partial_\mu J_\psi^\mu = \frac{2\alpha_Q + \alpha_U + \alpha_D}{2}\partial_\mu J_{\mathcal{PQ}}^\mu + \frac{\alpha_L + 3\alpha_Q}{2}\partial_\mu J_{\mathcal{B}+\mathcal{L}}^\mu + (\alpha_L + \alpha_E - \alpha_Q - \alpha_D)\partial_\mu J_E^\mu \ , \tag{3.10}$$

where we have used $\partial_\mu J_{\mathcal{B}-\mathcal{L}}^\mu = \partial_\mu J_Y^\mu = 0$. Plugging Eq. (3.8) for the divergences of the three currents, the variation $\Delta\mathcal{L}$ can be absorbed into shifts of the anomalous terms as

$$\theta_C \to \theta_C - N_f(2\alpha_Q + \alpha_U + \alpha_D) \ , \tag{3.11}$$

$$\theta_L \to \theta_L - N_f(\alpha_L + 3\alpha_Q) \ , \tag{3.12}$$

$$\theta_Y \to \theta_Y - \frac{1}{3}N_f(\alpha_Q + 8\alpha_U + 2\alpha_D + 3\alpha_L + 6\alpha_E) \ . \tag{3.13}$$

As could have been expected, $\theta_C$ is sensitive only to rotations of the quarks, $\theta_L$ to those of the weak doublets, and $\theta_Y$ to that of all the fermion fields. Also, without surprise, the anomalous shifts are invariant under $U(1)_Y$ and $U(1)_{\mathcal{B}-\mathcal{L}}$. As long as $G_F$ is exact, none of the anomalous terms is physical since for any $\theta_C$, $\theta_L$, $\theta_Y$ values, there exists a set of $U(1)$ transformations able to rotate them away.

The situation changes once $G_F$ is broken since these phase rotations are no longer all permitted. As a result, the Yukawa couplings and the $\theta$ terms are linked and cannot be defined separately. The usual convention is to ask for real and positive fermion mass terms. To achieve this, note first that the singular value decompositions which need to be done, Eq. (1.8), imply the $U(1)$ transformations $\alpha_{Q,L}N_f = \arg\det V_L^{d,e\dagger}$ and $\alpha_{U,D,E}N_f = \arg\det V_R^{u,d,e\dagger}$. At this level though, these phases are not entirely fixed because the SVD is ambiguous, and the various unitary matrices are defined up to a relative phase. So, to have real masses these phases must satisfy

$$\arg\det \mathbf{Y}_u = N_f(\alpha_Q + \alpha_U)\ ,\ \arg\det \mathbf{Y}_d = N_f(\alpha_Q + \alpha_D)\ ,\ \arg\det \mathbf{Y}_e = N_f(\alpha_L + \alpha_E)\ . \tag{3.14}$$

To reach this form, we used that $\arg\det \mathbf{m}_{u,d,e} = \arg\det V_{CKM} = 0$[1]. Eliminating $\alpha_{U,D,E}$, the shifts in the anomalous couplings generated when enforcing real fermion masses are

$$\theta_C \to \theta_C - \arg\det \mathbf{Y}_u - \arg\det \mathbf{Y}_d\ , \tag{3.15a}$$

$$\theta_L \to \theta_L - N_f\left(\alpha_L + 3\alpha_Q\right)\ , \tag{3.15b}$$

$$\theta_Y \to \theta_Y + N_f(\alpha_L + 3\alpha_Q) - \frac{1}{3}(8\arg\det \mathbf{Y}_u + 2\arg\det \mathbf{Y}_d + 6\arg\det \mathbf{Y}_e)\ . \tag{3.15c}$$

No choice of $\alpha_Q$ and $\alpha_L$ permits to remove both $\theta_L$ and $\theta_Y$. However, since the latter is harmless, we are free to chose $\alpha_L + 3\alpha_Q = \theta_L/N_f$ and remove the $SU(2)_L$ anomalous interactions. This freedom is clearly reminiscent of the invariance of $\mathcal{L}_{\text{Yukawa}}$ under the anomalous $U(1)_{\mathcal{B}+\mathcal{L}}$. Once this is done, there still remain a one-parameter freedom in the choice of the $\alpha'$s, this time reminiscent of the invariance under the non-anomalous $U(1)_{\mathcal{B}-\mathcal{L}}$.

In contrast to $\theta_L$, the requirement of real quark masses unambiguously freezes the $\theta_C$ anomalous interactions. This is the origin of the famous **strong $\mathcal{CP}$ puzzle** [11, 12]: experimentally, the non-observation of an electric dipole moment for the neutron rules out a significant $G_{\mu\nu}^a\tilde{G}^{a,\mu\nu}$ interaction,

$$\theta_{eff} = \theta_C - \arg\det \mathbf{Y}_u - \arg\det \mathbf{Y}_d \lesssim 10^{-10}\ . \tag{3.16}$$

But with $\theta_C$ originating from the non-perturbative vacuum of the $SU(3)_C$ gauge theory, and $\mathbf{Y}_{u,d}$ from the Higgs coupling to quarks, both are a priori $\mathcal{O}(1)$ and such a near-perfect cancellation is, to say the least, extremely puzzling.

There are two well-known ways out, though neither is entirely satisfactory yet. First, one of the quark mass could vanish. Though this is more or less ruled out experimentally, there are extensions of the SM where the lightest quarks could receive their masses from a different mechanism. But if $\det \mathbf{Y}_u$ or $\det \mathbf{Y}_d$ vanishes, then the corresponding constraint in Eq. (3.14) disappears since $\arg 0$ is a free number. This additional freedom permits to rotate $\theta_C$ away no matter its value. The second solution, first proposed by Peccei and Quinn [13], starts by promoting $U(1)_{\mathcal{PQ}}$ to an exact symmetry by assigning the adequate charges to the Higgs field. Since $U$ and $D$ have the same charges, this necessitates two Higgs doublets: one should give mass to the up quarks, and the other to the down quarks and leptons. This $U(1)_{\mathcal{PQ}}$ symmetry is then spontaneously broken, in such a way that the $\theta_{eff}$ relaxes towards zero. The price to pay is a new particle, the **axion** [14], as the Goldstone boson associated to the spontaneous breaking of $U(1)_{\mathcal{PQ}}$.

[1] Notice that rephasing of the CKM matrix are irrelevant here since they can be achieved rotating right and left-handed fields by opposite phases, see Eq. (1.11).

## 3.2 A flavored perspective on the proton stability

In the previous section, the theta term associated to the weak interaction was seen to be unphysical. Thanks to the $\mathcal{B}+\mathcal{L}$ anomaly, it can always be rotated away. This however does not mean that $\mathcal{B}$ and $\mathcal{L}$ violations are entirely disposed of. The situation in QCD perfectly illustrates this. Even if the strong $\mathcal{CP}$ puzzle is somehow resolved and $\theta_{eff}=0$ emerges naturally, the axial $G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}$ anomaly is still present, and ensures the $\eta'$ meson stays significantly more massive than the other pseudoscalar mesons. Intuitively, one can understand $\theta_C$ as originating from instanton transitions between vacua, while the $\eta'$ mass would result from such transitions between quark states. Similarly, the $W^i_{\mu\nu}\tilde{W}^{i,\mu\nu}$ coupling represents the vacuum structure of the $SU(2)_L$ gauge theory. It happens to be possible to rotate it away, but instantons can still induce $\mathcal{B}+\mathcal{L}$ transitions between fermion states.

The QCD axial anomaly and the $\mathcal{B}+\mathcal{L}$ anomaly are both breaking some flavor $U(1)$s but not any of the associated $SU(3)$s. As far as transitions among generations are concerned, these interactions are perfectly blind. Introducing a slight misnomer, we thus call them **flavor-blind interactions**. In practice, the only way a flavor-blind interaction could have an axial or $\mathcal{B}+\mathcal{L}$ charge is by involving contractions of the fermion fields with the Levi-Civita invariant tensor of the $SU(3)$ flavor groups. It is then automatically invariant under $SU(3)^5$, and explicitly breaks the $U(1)$ associated to the epsilon tensor[2]. For example:

$$\varepsilon^{IJK}Q^IQ^JQ^K \to \varepsilon^{IJK}(g_QQ)^I(g_QQ)^J(g_QQ)^K = \det(g_Q)\varepsilon^{IJK}Q^IQ^JQ^K \;, \tag{3.17}$$

since for any matrix, $\varepsilon^{IJK}X^{IL}X^{JM}X^{KN}=\det(X)\varepsilon^{LMN}$. Obviously, any interaction must involve an even number of such factors to respect Lorentz invariance.

Still guided only by the flavor symmetry, let us try to construct the effective interactions generated by the axial and $\mathcal{B}+\mathcal{L}$ anomalies. Both should be expressed as $SU(3)_C\otimes SU(2)_L\otimes U(1)_Y$ invariant couplings among fermion fields, manifestly symmetric under $SU(3)^5$ but carrying the appropriate overall $U(1)$ charge. For the axial anomaly which involves only quark fields, the simplest such interaction requires 12 quarks

$$\mathcal{H}^{axial}_{eff}\sim\frac{g^{axial}}{M_W^{14}}(\varepsilon^{IJK}Q^IQ^JQ^K)^2(\varepsilon^{IJK}U^IU^JU^K)(\varepsilon^{IJK}D^ID^JD^K)+h.c.\;, \tag{3.18}$$

where the $SU(2)_L$, $SU(3)_C$, and Lorentz spinorial contractions are understood (only those contractions that maximally entwine the antisymmetric tensors do not vanish identically). Note that under a $U(1)_Q\otimes U(1)_U\otimes U(1)_D$ transformation, the phase of this coupling is shifted by $g^{axial}\to g^{axial}\exp iN_f(2\alpha_Q+\alpha_U+\alpha_D)$, exactly like the strong $\theta_C$ in Eq. (3.11). Indirectly, $\theta_C$ is thus in principle measurable since with $\arg(g^{axial})$ presumably vanishing, the phase of this twelve-quark interaction directly measure $\arg\det\mathbf{Y}_u+\arg\det\mathbf{Y}_d$.

Actually, we can recognize in this interaction the 't Hooft determinant [15]. If the $SU(2)_L$ contractions are expanded, it collapses to a product of $\bar{\psi}_R\psi_L$ factors over the six flavors of quarks $\psi=u,c,t,d,s,b$. In this respect, it is amusing to remember that the 't Hooft determinant originates as a fermionic Gaussian path integration. At the level of the flavor symmetry, it is simply the only structure [84] breaking $U(3)$ down to $SU(3)$. Finally, it should be noted that this interaction is not particularly interesting phenomenologically because it involves too many quark fields. On the other hand, at low energy in a restricted world with only the three light quark flavors $u$, $d$, and $s$, the 6-quark effective interaction is thought to non-perturbatively generate the large $\eta'$ mass, and thus solves the $U(1)_A$ puzzle of low-energy QCD.

[2] To our knowledge, the first appearance of epsilon tensors in a MFV context was in Ref. [83], though the $SU(3)$-symmetric aspect of the anomalous interactions were already elucidated in Ref. [84].

The same strategy can be applied to write down the effective interactions among fermions induced by the $\mathcal{B}+\mathcal{L}$ anomaly. Again, no flavor-blind $SU(3)_C\otimes SU(2)_L\otimes U(1)_Y$ invariant interaction violating $\mathcal{B}$ and/or $\mathcal{L}$ can be constructed with six fields, so at least twelve fermions are needed. By inspection, four such interactions can be found (these can also be guessed directly from Eq. (3.1)):

$$\mathcal{H}_{eff} \sim \frac{1}{M_W^{14}}(g_1^{\mathcal{B}+\mathcal{L}} L^3 Q^9 + g_2^{\mathcal{B}+\mathcal{L}} L^3 Q^3 D^{\dagger 3} U^{\dagger 3} + g_3^{\mathcal{B}+\mathcal{L}} E^{\dagger 3} U^{\dagger 6} D^{\dagger 3} + g_4^{\mathcal{B}+\mathcal{L}} E^{\dagger 3} Q^6 U^{\dagger 3}) \,, \tag{3.19}$$

where the four epsilon contractions in flavor space are understood (those are similar as in Eq. (3.18)), as well as the color, $SU(2)_L$, and spinor contractions. These interactions violate $\mathcal{B}$ and $\mathcal{L}$ by three units, but respect $\mathcal{B}-\mathcal{L}$. From a flavored $U(1)$ perspective, the first one transforms precisely as the $SU(2)_L$ anomaly, i.e.,

$$g_1^{\mathcal{B}+\mathcal{L}} \to g_1^{\mathcal{B}+\mathcal{L}} \exp iN_f(3\alpha_Q + \alpha_L) \,, \tag{3.20}$$

see Eq. (3.12), and actually corresponds to the true $U(1)_{\mathcal{B}+\mathcal{L}}$ anomalous interactions. The others are not generated by the SM dynamics since $W$ bosons do not couple to right-handed fermions. Though this construction does not single out only the correct $U(1)_{\mathcal{B}+\mathcal{L}}$ interaction, it quite impressively suffices to predict that the simplest $\mathcal{B}$ and/or $\mathcal{L}$ violating interaction that could ever be flavor-blind preserve $\mathcal{B}-\mathcal{L}$, and has to involve twelve fields. In this respect, it should be clear that different flavor-blind patterns of $\mathcal{B}$ and $\mathcal{L}$ violation are possible but require at least six more fermion fields. For instance, with 18 fermions, $\pm(\Delta\mathcal{B},\Delta\mathcal{L}) = (6,0)$, $(0,6)$, $(3,9)$, or $(3,\pm 3)$ interactions can be written down.

None of these interactions induce easily observable processes like proton decay. In addition, the anomalous interaction is extremely suppressed in the SM because $g_1^{\mathcal{B}+\mathcal{L}} \sim \exp(-8\pi^2/g^2) \sim 10^{-180}$. So, they are irrelevant phenomenologically in particle physics experiments[3]. However, they could play a crucial role for cosmology. Indeed, though the strength of the anomalous interaction does not increase at high energy, it is not so at high temperature. There, gauge field configurations called **sphalerons** are able to copiously induce the $\mathcal{B}+\mathcal{L}$ transition. So, if for some reasons the universe develops an $\mathcal{L}$ asymmetry in its early stages, then sphalerons can transform it into the observed $\mathcal{B}$ asymmetry. This is the basic idea of the **leptogenesis** scenario [81].

Before closing this section, we can summarize the characteristics of the flavor-blind $\mathcal{B}$ and/or $\mathcal{L}$ violating interactions by stating two generic properties:

1. **Selection rule**: Because fermion fields have to be contracted by epsilon tensors in flavor space, $\mathcal{B}$ and $\mathcal{L}$ have to be violated in steps of $N_f$ elementary units. At the same time, the elementary unit of $\mathcal{B}$ carried by quarks is $1/N_C$, with $N_C$ the number of QCD colors, since the colorless proton made of $N_C$ quarks contracted together by the $SU(N_C)$ epsilon tensor has, by convention, $\mathcal{B}=+1$. So, $\mathcal{B}$ and/or $\mathcal{L}$ violation must fulfill:

$$1/N_f \times \Delta\mathcal{L} \in \mathbb{Z}, \quad N_C/N_f \times \Delta\mathcal{B} \in \mathbb{Z} \,. \tag{3.21}$$

   For most values of $N_f$ and $N_C$, the smallest $\Delta\mathcal{B}$ is simply the number of flavors $N_f$. The real world situation with $N_C = N_f$ is actually the least constraining situation for $\mathcal{B}$ violation, since the flavor contraction is redundant with color invariance in this case.

2. **Flavor-democracy**: Intuitively, a flavor-blind combination of fermion fields is either of the form $\delta^{IJ}\bar{f}^I f^J$ (scalar product) or $\varepsilon^{IJK} f^I f^J f^K$ (cross product). Only the latter can carry a $U(1)$ charge. Phenomenologically, the most important feature of the epsilon tensor is its antisymmetry: all three flavors of fermions are represented, including the heaviest ones.

[3] This is quite unfortunate since, as for the axial anomaly, a measurement of the phase of $g_1^{\mathcal{B}+\mathcal{L}}$ indirectly gives $\theta_L$ [85], see Eqs. (3.20) and (3.15).

| $\Delta\mathcal{L}$ | $\Delta\mathcal{B}$ | Core interactions | Largest couplings |
|---|---|---|---|
| $\pm 6$ | 0 | $\nu\nu\nu\otimes\nu\nu\nu$ | $\nu_e\,\nu_\mu\,\nu_\tau\otimes\nu_e\,\nu_\mu\,\nu_\tau$ |
| $\pm 3$ | $\pm 1$ | $uuu\otimes\ell\ell\nu$ | $t\,c\,u\otimes e^-\,\mu^-\,\nu_\tau$ |
| | | $uud\otimes\ell\nu\nu$ | $t\,c\,d\otimes e^-\,\nu_\mu\,\nu_\tau$ |
| | | $udd\otimes\nu\nu\nu$ | $t\,s\,d\otimes\nu_e\,\nu_\mu\,\nu_\tau$ |
| $\pm 3$ | $\mp 1$ | $udd\otimes\bar\nu\bar\nu\bar\nu$ | $t\,s\,d\otimes\bar\nu_e\,\bar\nu_\mu\,\bar\nu_\tau$ |
| | | $ddd\otimes\bar\ell\bar\nu\bar\nu$ | $b\,s\,d\otimes e^+\,\bar\nu_\mu\,\bar\nu_\tau$ |
| 0 | $\pm 2$ | $udd\otimes udd$ | $t\,s\,d\otimes t\,s\,d$ |
| | | $uud\otimes ddd$ | $t\,c\,d\otimes b\,s\,d$ |

Table 3.1: Generic particle content of the $\Delta\mathcal{B}$ and $\Delta\mathcal{L}$ six-fermion interactions (charge-conjugate interactions are understood). The last column shows one dominant flavor assignment for each interaction.

### 3.2.1 Flavor-breaking $\mathcal{B}$ and $\mathcal{L}$ violation

In the SM, the $SU(3)^5$ symmetry is broken explicitly, so one may wonder whether the anomaly indeed represents the only possible $\mathcal{B}$ and/or $\mathcal{L}$ violating interaction. What we will show here is that simpler $\mathcal{B}$ and/or $\mathcal{L}$ violating interactions are possible, though the anomaly remains the simplest $\mathcal{B}+\mathcal{L}$ violating interaction. Of course, the SM dynamics does not generate these simpler interactions, but they could naturally arise from the physics beyond the SM since they are compatible with the available flavor structures.

Once the SM Yukawa spurions are introduced, there are three modifications allowing for interactions with less than twelve fermion fields:

1. The selection rule Eq. (3.21) remains exact but the flavor content may be altered. This is sensible since the flavor of quark fields is not preserved in the SM.
2. Fermions of different kinds can be contracted together at the cost of some spurion insertions. For example $\varepsilon^{IJK}Q^IQ^J(D\mathbf{Y}_d)^{\dagger K}$ is invariant under $SU(3)^5$.
3. The Higgs field doublet carry an hypercharge, hence can be used to construct additional gauge invariant combinations of fermion fields.

Since an even number of fermion fields is required by Lorentz invariance, the only interactions simpler than those obtained in the previous section are those with six fermions. By an explicit and systematic search, including any number of Higgs fields, the whole set of six-fermion interactions can be written down. Four possible patterns of $\mathcal{B}$ and $\mathcal{L}$ violation with six fermions emerges, $|\Delta\mathcal{L}|=6$, $\Delta\mathcal{L}=-3\Delta\mathcal{B}$, $\Delta\mathcal{L}=+3\Delta\mathcal{B}$, and $|\Delta\mathcal{B}|=2$, with those operators involving the least number of Higgs boson fields (see Appendix A.2 for the full list) being [27]

$$\mathcal{H}_{eff}=\frac{H^6L^6}{\Lambda^{11}}+\frac{HL^3D^3}{\Lambda^6}+\frac{EL^{\dagger 2}U^3+L^{\dagger 3}Q^\dagger U^2}{\Lambda^5}+\frac{U^2D^4+Q^{\dagger 2}UD^3+Q^{\dagger 4}D^2}{\Lambda^5}+h.c.\,, \tag{3.22}$$

respectively. For simplicity, the flavor contractions as well as the various possible spurion insertions are not written explicitly. Numerical $\mathcal{O}(1)$ coefficients are also understood, while $\Lambda$ represents the typical energy scale of the process generating these non-renormalizable interactions. Not all the interactions above require Yukawa spurions to be invariant under $SU(3)^5$, but whenever they do not, Higgs fields are present.

Phenomenologically, the six-fermion interactions could have interesting signatures [29]. Let us concentrate on the simplest interactions, without Higgs fields. The largest couplings are always

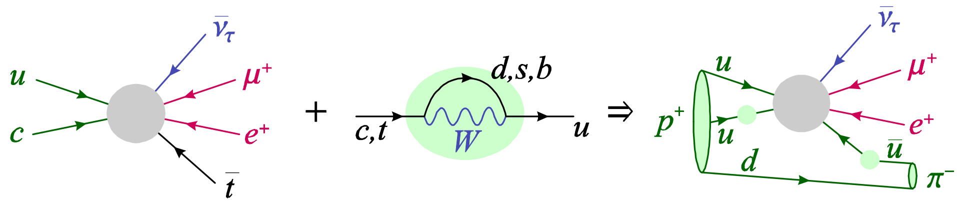


Figure 3.2: Dressing of the dominant dimension-nine effective coupling by flavor-changing effects, leading to a proton-decay inducing interaction.

those involving three different generations of quarks and leptons. This is due to the antisymmetric contraction. Examples of such flavor-antisymmetric six-fermion channels are listed in Table 3.1. Experimentally, the most accessible channels are those with the least number of neutrinos, simply because the kinematics, the lepton number, and the flavor of these particles cannot be measured. Two channels are thus particularly promising: the $\Delta\mathcal{L} = 3\Delta\mathcal{B}$ interaction like $t\,c\,u \otimes e^-\,\mu^-\,\nu_\tau$, and the $\Delta\mathcal{B} = 2$ interaction like $t\,s\,d \otimes t\,s\,d$. In both cases, the final state can contain a characteristic pair of leptons of the same sign. In Section 3.3, we will see that the MSSM with $\mathcal{R}$-parity violation is a specific dynamical realization for the $\Delta\mathcal{B} = 2$ interaction, and its same-sign lepton pair signature will be discussed in details.

Though the largest couplings always involve heavy particles, the $\Delta\mathcal{L} = 3\Delta\mathcal{B}$ and $\Delta\mathcal{B} = 2$ interactions can nevertheless contribute at low-energy, see Fig. 3.2. This is particularly worrisome for the former, since it can induce proton decay. But, because of the flavor symmetry requirement, these interactions exhibit a strong hierarchy in flavor space, inherited from the Yukawa couplings. Specifically, only interactions with light-quark flavors can contribute to proton decay. So, one must rely on insertions of $\mathbf{Y}_u$ (if we use the basis Eq. (1.21)), and go pick its CKM-suppressed non-diagonal entries. An example can be useful. Take the $L^{\dagger 3}Q^\dagger U^2$ interaction and extract its $d_L u_R u_R$ piece:

$$\varepsilon^{IJK} Q^I (U\mathbf{Y}_u)^{\dagger J} (U\mathbf{Y}_u)^{\dagger K} \ni d_L u_R u_R \times (\mathbf{Y}_u^\dagger)^{12} (\mathbf{Y}_u^\dagger)^{13} = d_L u_R u_R \times \left(\frac{m_u}{v_u}\right)^2 V_{us}^\dagger V_{ub}^\dagger \,. \tag{3.23}$$

Numerically, this is a suppression by $(m_u/v_u)^2 V_{us}^\dagger V_{ub}^\dagger \sim 10^{-14}$ compared to the leading interaction $d_L c_R t_R$, suppressed only by $m_c/v_u$. Similarly, the light-quark terms in the $EL^{\dagger 2}U^3$ operator are suppressed by $(m_u/v_u)^3 V_{us} V_{ub} \sim 10^{-19}$. This flavor suppression combined with the overall factor of the order of $\mathcal{O}(m_{p^+}^{11}/\Lambda^{10})$ for the decay rate, ensures the proton lifetime is above about $10^{30}$ years even for a relatively low scale, $\Lambda \gtrsim 1$ TeV. This is sufficient since the bounds [17] on the $\Delta\mathcal{L} = \pm 3$ decay channels are much less tight than the best bound on the $\Delta\mathcal{L} = 1$ mode $p^+ \to e^+\pi^0$, which cannot be induced by $L^{\dagger 3}Q^\dagger U^2$ or $EL^{\dagger 2}U^3$. These two suppression mechanisms similarly ensure that the $\Delta\mathcal{B} = 2$ operators do not induce too rapid $n - \bar{n}$ oscillations [27].

### 3.2.2 $\mathcal{B}$ and $\mathcal{L}$ violation and the seesaw mechanism

In the presence of the right-handed neutrinos, $\mathcal{L}$ is no longer naturally conserved since they could have a $\Delta\mathcal{L} = 2$ Majorana mass term. The $\Delta\mathcal{L} = N_f \times \mathbb{Z}$ selection rule discussed above disappears, and consequently, the pattern of $\mathcal{B}$ and/or $\mathcal{L}$ violation is deeply altered. To analyze this situation, let us treat the effective left-handed $\Delta\mathcal{L} = 2$ Majorana mass term as part of the spurions. Then, we can search again for the simplest interactions invariant under $SU(3)^5$, but carrying an overall $\mathcal{B}$ and/or $\mathcal{L}$ charge. Clearly, $\mathcal{B}$ violation still requires multiples of three quarks because of color invariance, but there is no restriction on the number of lepton fields. The simplest interactions are thus directly those proposed by Weinberg [78] (and later extended in Ref. [86]),

$$\mathcal{H}_{eff} = \frac{H^2 LL}{\Lambda} + \frac{LQQQ + LQD^\dagger U^\dagger + E^\dagger U^\dagger U^\dagger D^\dagger + E^\dagger U^\dagger QQ}{\Lambda^2} \tag{3.24}$$

$$+ H^\dagger \frac{L^\dagger D^\dagger D^\dagger D^\dagger + LUDD + LDQ^\dagger Q^\dagger + E^\dagger Q^\dagger DD}{\Lambda^3} + h.c. \,, \tag{3.25}$$

which induce $\pm(\Delta\mathcal{B}, \Delta\mathcal{L}) = (0, 2)$, $(1, 1)$, or $(1, -1)$ transitions, respectively. Other interactions with two or more Higgs fields can be constructed, but they all induce the same $(\Delta\mathcal{B}, \Delta\mathcal{L})$ patterns. Intriguingly, the Higgsless $(\Delta\mathcal{B}, \Delta\mathcal{L}) = (1, 1)$ interactions violate $\mathcal{B}+\mathcal{L}$ but respect $\mathcal{B}-\mathcal{L}$, exactly like the $SU(2)_L$ anomaly, Eq. (3.19). From a flavor point of view, these dimension-six interactions can be made to break the same $U(1)_Q$ and $U(1)_L$ as the $SU(2)_L$ anomaly by introducing the appropriate spurions, hence cannot be ruled out on that basis.

There are two main differences with the anomalous interactions though. First, dimension-six interactions induce proton decay, so their strength is tightly constrained experimentally. Second, they are never flavor-blind, hence require non-trivial spurion insertions to be invariant under $SU(3)^5$. For example, from Ref. [27], the leading contribution is

$$\frac{1}{\Lambda^2} LQQQ \to \frac{1}{\Lambda^2} \varepsilon^{IMN} (\mathbf{\Upsilon}_\nu \mathbf{Y}_e^\dagger \mathbf{Y}_e)^{MN} L^I \times \varepsilon^{JKO} (\mathbf{Y}_u^\dagger \mathbf{Y}_u)^{OL} Q^J Q^K Q^L \,, \tag{3.26}$$

where in the basis where $v\mathbf{Y}_e = \mathbf{m}_e$, $v\mathbf{\Upsilon}_\nu \equiv U^*_{PMNS} \mathbf{m}_\nu U^\dagger_{PMNS}$, $\mathbf{m}_\nu = \mathrm{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3})$, and $U_{PMNS}$ the neutrino mixing matrix. This renders the couplings highly hierarchical in flavor space, and forces them to involve the neutrino masses $m_\nu/v \sim 10^{-9}$. Numerically, these suppressions permits to pass proton decay bounds even with $\Lambda$ close to the TeV scale [27] (see also Ref. [87]), but rules out any visible effect at colliders [88].

## 3.3 A new $\mathcal{B}$-violating supersymmetric paradigm

With the SM particle content, the simplest $\mathcal{B}$ or $\mathcal{L}$ violating couplings involve four or more fermions, since only fermions carry these quantum numbers. Those couplings are thus at least of dimension six and are discarded by invoking renormalizability. In the MSSM, the presence of the flavored scalar partners of the quarks and leptons, with identical internal quantum numbers, allows us to build renormalizable couplings violating $\mathcal{B}$ or $\mathcal{L}$. Specifically, given the gauge quantum numbers of matter fields, more terms are immediately permitted in the superpotential [89],

$$\mathcal{W}^{\Delta\mathcal{B},\Delta\mathcal{L}}_{\text{Yukawa}} = \mu'^I L^I H_u + \frac{1}{2} \mathbf{Y}^{IJK}_{eee} L^I L^J E^K + \mathbf{Y}^{IJK}_{eud} L^I Q^J D^K + \frac{1}{2} \mathbf{Y}^{IJK}_{udd} U^I D^J D^K \,, \tag{3.27}$$

where $I, J, K = 1, 2, 3$ denote flavor indices. The first three interactions violate $\mathcal{L}$, the last one $\mathcal{B}$. Similar terms occur also among the soft-breaking terms

$$\mathcal{L}^{\Delta\mathcal{B},\Delta\mathcal{L}}_{\text{Soft}} = -\mathbf{b}'^I (H_u \tilde{L}^I) - (\mathbf{m}^2_{Ld})^I H_d^\dagger \cdot \tilde{L}^I + h.c. \tag{3.28}$$

$$+ \frac{1}{2} \mathbf{A}^{IJK}_{eee} \tilde{L}^I \tilde{L}^J \tilde{E}^K + \mathbf{A}^{IJK}_{eud} \tilde{L}^I \tilde{Q}^J \tilde{D}^K + \frac{1}{2} \mathbf{A}^{IJK}_{udd} \tilde{U}^I \tilde{D}^J \tilde{D}^K + h.c. \,, \tag{3.29}$$

which involve only the scalar fields. Due to the antisymmetric $SU(2)_L$ contraction, the $\mathbf{Y}^{IJK}_{eee}$ and $\mathbf{A}^{IJK}_{eee}$ couplings are antisymmetric under $I \leftrightarrow J$. Similarly, $\mathbf{Y}^{IJK}_{udd}$ and $\mathbf{A}^{IJK}_{udd}$ are antisymmetric under $J \leftrightarrow K$ due to the antisymmetric contraction of color indices with $\varepsilon^{abc}$. Altogether, these couplings represent about 100 (a priori complex) additional free parameters.

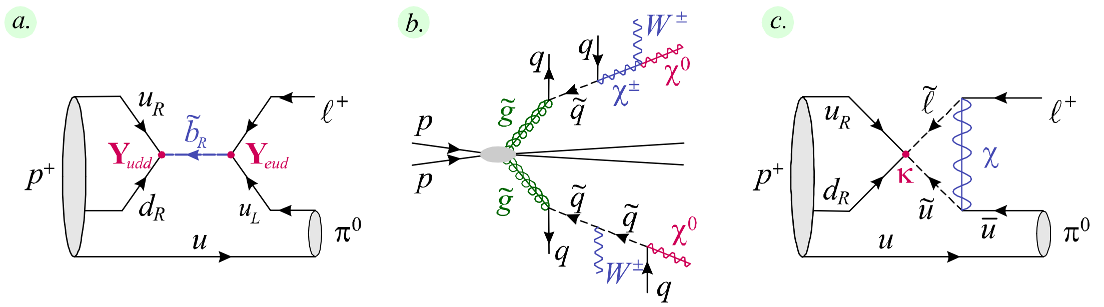


Figure 3.3: ($a$) Tree-level decay of the proton induced by the $\mathcal{R}$-parity violating couplings. ($b$) Typical LHC event in the $\mathcal{R}$-parity conserving MSSM: sparticles, initially produced in pairs, cascade-decay into neutral and stable LSPs which then escape the detector. ($c$) Example of proton decay mechanism induced by dimension-five $\mathcal{R}$-parity conserving operators at the loop-level.

### 3.3.1 Why is the MSSM plagued by $\mathcal{R}$-parity?

Let us concentrate on $\mathcal{W}_{\text{Yukawa}}^{\Delta\mathcal{B},\Delta\mathcal{L}}$, from which the following Yukawa couplings can be derived:

$$\mathcal{L}_{\text{Yukawa}}^{\Delta\mathcal{B},\Delta\mathcal{L}} = -\frac{1}{2}\mathbf{Y}_{eee}^{IJK}(L^I L^J \tilde{E}^K + 2\tilde{L}^I L^J E^K) - \mathbf{Y}_{eud}^{IJK}(L^I Q^J \tilde{D}^K + L^I D^K \tilde{Q}^J + \tilde{L}^I Q^J D^K) \\ - \mu'^I \tilde{H}_u L^I - \frac{1}{2}\mathbf{Y}_{udd}^{IJK}\varepsilon^{abc}(D_a^J D_b^K \tilde{U}_c^I + 2U_a^I D_b^J \tilde{D}_c^K) + h.c. \; . \tag{3.30}$$

These interactions induce proton decay at tree level, see Fig. 3.3$a$, and are thus severely constrained experimentally

$$\Gamma_p \approx \frac{\alpha^2 m_p}{4\pi F_\pi^2}\frac{|\mathbf{Y}_{eud}^\dagger \mathbf{Y}_{udd}|^2}{M_{\tilde{d}}^4} \rightarrow |\mathbf{Y}_{eud}^\dagger \mathbf{Y}_{udd}| \lesssim 10^{-27}\left(\frac{M_{\tilde{d}}}{100\,\text{GeV}}\right)^2 \; . \tag{3.31}$$

This bound can only be natural for squark mass around $10^{15}$ GeV, that is, at around the GUT scale. This is no surprise since integrating out the squarks reproduces some of the Weinberg operators of Eq. (3.1). On the other hand, if squarks are to be around the TeV scale, so as to help alleviate the hierarchy puzzle, this bound is so strict that introducing a symmetry forbidding proton decay appears compulsory.

The common choice is to supplement the MSSM with the so-called $\mathcal{R}$-parity [90], defined as $\mathcal{R} = (-1)^{2S}(-1)^{3(\mathcal{B}-\mathcal{L})}$ where $S$ is the spin. In the SM, gauge bosons have $(\mathcal{B},\mathcal{L},S) = (0,0,1)$, Higgs bosons have $(0,0,0)$, quarks have $(1/3,0,1/2)$, and leptons have $(0,1,1/2)$. They are thus all even under $\mathcal{R}$-parity. On the other hand, their superpartners are odd since they carry the same $\mathcal{B}$ and $\mathcal{L}$ but have a spin differing by $\pm 1/2$. Clearly, all the couplings derived from $\mathcal{W}_{\text{Yukawa}}^{\Delta\mathcal{B},\Delta\mathcal{L}}$, as well as those in $\mathcal{L}_{\text{Soft}}^{\Delta\mathcal{B},\Delta\mathcal{L}}$, are strictly forbidden if $\mathcal{R}$-parity is conserved.

Imposing $\mathcal{R}$-parity on the MSSM drastically alters its phenomenology, but given the seriousness of the proton decay puzzle, this is generally accepted at least as a working assumption. The most obvious impacts are that superpartners are always produced in pairs, and that the lightest supersymmetric particle (LSP) is absolutely stable. Therefore, to get a significant signal of supersymmetry at the LHC typically requires the center-of-mass energy to be comparable to twice the supersymmetry mass scale. The two produced sparticles then cascade decay down to a bunch of LSPs which escape undetected (missing energy), see Fig. 3.3$b$. The only "beneficial" aspect of $\mathcal{R}$-parity, apart of course from proton decay and from the reduction in the number of free parameters[4], is that the LSP may be a good dark

[4]Allowing for the $\Delta\mathcal{L}$ interactions is nevertheless welcome in some scenario as a way to induce small neutrino

matter candidate, provided it is neutral and colorless. It is therefore often assumed that one of the neutralino is the lightest superparticle. Then, it remains to be seen if it can be produced in the right amount to fit astrophysical observations.

In the following, our goal will be to show that the $\mathcal{R}$-parity violating couplings are actually naturally small, in the sense that imposing MFV on them is sufficient to pass all proton decay bounds. They do not require particular fine tuning, besides those already observed in the strange pattern of quark and lepton masses and mixings. Therefore, $\mathcal{R}$-parity looses its main appeal, and should no longer be imposed on the MSSM. Indeed, it seems not reasonable to impose such drastic changes on the phenomenology only for the quite indirect purpose of fitting dark matter constraints, especially given the fact that we know the MSSM cannot be the ultimate theory.

From a more theoretical point of view, $\mathcal{R}$-parity is usually not believed to be a perfect solution for proton decay anyway. Since the MSSM lacks a dynamical mechanism to break supersymmetry, does not explain the origin of the flavor structures, and must be extended to naturally account for tiny neutrino masses, it most certainly gets supplanted by another model at some scale. This would be felt at low energy through effective, non-renormalizable interactions. Let us thus consider the following $\mathcal{R}$-parity even $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant dimension-five operators [92]:

$$\mathcal{W}_{\dim-5} = \frac{\boldsymbol{\kappa}_1^{IJKL}}{\Lambda}(Q^I Q^J)(Q^K L^L) + \frac{\boldsymbol{\kappa}_2^{IJKL}}{\Lambda}(D^I U^J U^K)E^L + \frac{\boldsymbol{\kappa}_5^{IJ}}{\Lambda}(L^I H_u)(L^J H_u) \ . \tag{3.32}$$

The last term is just the Majorana mass with $\Lambda$ the seesaw scale $M_R$. The similarity with Eq. (3.1) is obvious, but remember that these are superpotential terms, hence their interactions are to be found by taking derivatives, and this yields dimension-five two-scalar– two-fermion interactions. That is why now only one power of the scale $\Lambda$ appears in the denominators. The important point is that these effective interactions can induce proton decay, even though they conserve $\mathcal{R}$-parity. Their contribution is necessarily loop level, since again they are $\mathcal{R}$-parity conserving (see Fig. 3.3*c*), so they depend on the other supersymmetric particle masses and couplings. As a naive estimate, we get

$$\Gamma_{p^+} \approx \frac{\alpha_p^2 m_p}{4\pi F_\pi^2}\frac{1}{M_{SUSY}^2}\frac{|\kappa_{1,2}|^2}{\Lambda^2} \lesssim 10^{-62}\text{ GeV} \Rightarrow \begin{cases} \Lambda \gtrsim 10^{27}\text{ TeV} & \text{if } \kappa_{1,2} \sim \mathcal{O}(1)\ , \\ \Lambda \gtrsim \Lambda_{Planck} & \text{if } \kappa_{1,2} \sim \mathcal{O}(10^{-8})\ , \\ \Lambda \lesssim 1\text{ TeV} & \text{if } \kappa_{1,2} \sim \mathcal{O}(10^{-25})\ , \end{cases} \tag{3.33}$$

where $M_{SUSY} \sim 1$ TeV stands for a common mass scale for supersymmetric particles. Even at the Planck scale, these operators must be extremely small. The trivial way out seems to be, once again, to send both $M_{SUSY}$ and $\Lambda$ above $10^{15}$ GeV. Here, we will of course call in MFV to suppress these operators, and we already know this works very effectively since the similar Weinberg operators were considered in the previous section.

### 3.3.2 $\mathcal{R}$-parity violation under MFV

When MFV is enforced, the $\mathcal{R}$-parity violating couplings should be expressed as expansions in powers of the Yukawa couplings. As explained before, this does not mean the former are less fundamental than the latter. Rather, the assumption is that both derive from a minimal set of fundamental spurions. It is their redundancy which is expressed by writing down the MFV expansions.

The crucial observation on which the whole viability of MFV rests is the incompatibility of the lepton number violating couplings with the selection rule Eq. (3.21). With only $\mathbf{Y}_e$ in the lepton

masses [91]. This can be understood from the fact that once $\mathcal{L}$ is no longer a good quantum numbers, the lepton doublet $L$ and the Higgsino doublet $\tilde{H}_d$ mix. The neutrino mass eigenstates are then massive due to a small neutralino admixture. The main issue in these scenarios is to control these mixings. Given the large mass splitting between neutralinos and neutrinos, this requires delicate and not very appealing fine-tunings of the $\Delta\mathcal{L} = 1$ couplings.

sector, there is no way to construct the $\mu'$, $\mathbf{Y}_{eee}$, or $\mathbf{Y}_{eud}$ couplings (or the $\mathcal{R}$-parity conserving $\boldsymbol{\kappa}_{1,2,3}$ effective couplings), and lepton number becomes exactly conserved. The proton is then stable, and only baryon number violation should be expected. Of course, as discussed in the context of the Weinberg operators, neutrinos do have masses. When those are of the Dirac type, the $\Delta\mathcal{L} = 1$ couplings are still impossible to construct. They become permitted only in the presence of a $\Delta\mathcal{L} = 2$ seesaw mechanism. But, being proportional to neutrino masses, they are sufficiently tiny to allow for sparticles at around the TeV scale without violating proton decay bounds. The analysis in this case is presented in Ref. [25], and will not be detailed here because it would require us to delve too far into the neutrino phenomenology.

### MFV expansions

Let us now construct the expansions for the $\mathcal{B}$ violating couplings. Given that a generic $\mathbf{Y}_{udd}$ introduces nine arbitrary complex parameters, the simplest polynomial expansions require nine independent terms. The strategy to chose them is to first consider possible contractions with epsilon tensors. This step was described in Ref. [25]. Here, we consider only the three simplest epsilon structures[5]

$$(\mathbf{Y}_{udd}^{Q})^{IJK} \sim \varepsilon^{LMN}\mathbf{Y}_u^{IL}\mathbf{Y}_d^{JM}\mathbf{Y}_d^{KN} + \ldots\ , \tag{3.34a}$$

$$(\mathbf{Y}_{udd}^{D})^{IJK} \sim \varepsilon^{LJK}(\mathbf{Y}_u\mathbf{Y}_d^{\dagger})^{IL} + \ldots\ , \tag{3.34b}$$

$$(\mathbf{Y}_{udd}^{U})^{IJK} \sim \varepsilon^{IMN}(\mathbf{Y}_d\mathbf{Y}_u^{\dagger})^{JM}(\mathbf{Y}_d\mathbf{Y}_u^{\dagger})^{KN} + \ldots\ , \tag{3.34c}$$

where either the epsilon tensor of $SU(3)_Q$, $SU(3)_D$, or $SU(3)_U$ is used. From this, the most general expansions are obtained by inserting in all possible ways the $SU(3)_Q$ octet expansions of Eq. (2.17). At this stage, because of the epsilon contractions, some redundant terms remain. The final step is to remove them and identify the minimal set of nine independent terms using the matrix identities derived in Appendix A.3, which combine Cayley-Hamilton theorem with the definition of the determinant.

These identities permit to get rid of many terms. Take for example the $\varepsilon^{LMN}\mathbf{Y}_u^{IL}\mathbf{Y}_d^{JM}\mathbf{Y}_d^{KN}$ structure. Any $SU(3)_Q$ octet insertion acting on $\mathbf{Y}_d$ can be moved to act on $\mathbf{Y}_u$ using either Eq. (A.33a),

$$\varepsilon^{LMN}\mathbf{Y}_u^{IL}[(\mathbf{Y}_d\mathbf{O})^{JM}\mathbf{Y}_d^{KN} + \mathbf{Y}_d^{JM}(\mathbf{Y}_d\mathbf{O})^{KN}] = \varepsilon^{LMN}(\mathbf{Y}_u\,[\langle\mathbf{O}\rangle - \mathbf{O}])^{IL}\mathbf{Y}_d^{JM}\mathbf{Y}_d^{KN}\ , \tag{3.35}$$

where the two terms on the left-hand side enforce $\mathbf{Y}_{udd}^{IJK} = -\mathbf{Y}_{udd}^{IKJ}$, or Eq. (A.35),

$$\varepsilon^{LMN}\mathbf{Y}_u^{IL}(\mathbf{Y}_d\mathbf{O})^{JM}(\mathbf{Y}_d\mathbf{O})^{KN} = \varepsilon^{LMN}(\mathbf{Y}_u[\mathbf{O}^2 - \langle\mathbf{O}\rangle\mathbf{O} + \tfrac{1}{2}\langle\mathbf{O}\rangle^2 - \tfrac{1}{2}\langle\mathbf{O}^2\rangle])^{IL}\mathbf{Y}_d^{JM}\mathbf{Y}_d^{KN}\ , \tag{3.36}$$

where $\mathbf{O}$ is an arbitrary complex matrix. The right-hand side retains a manifestly $SU(3)_Q$ invariant form since $\mathbf{O}$ transforms as an octet. Therefore, octets need to act on the $\mathbf{Y}_u$ factor only, and the final set of nine terms can be chosen as ($\mathbf{X}_{u,d} \equiv \mathbf{Y}_{u,d}^{\dagger}\mathbf{Y}_{u,d}$)

$$\begin{aligned}(\mathbf{Y}_{udd}^{Q})^{IJK} = \varepsilon^{LMN}(\mathbf{Y}_u(&\lambda_1^q\mathbf{1} + \lambda_2^q\mathbf{X}_u + \lambda_3^q\mathbf{X}_d + \lambda_4^q\mathbf{X}_u^2 + \lambda_5^q\mathbf{X}_d^2 + \lambda_6^q\{\mathbf{X}_u,\mathbf{X}_d\} \\ &+ \lambda_7^q i[\mathbf{X}_u,\mathbf{X}_d] + \lambda_8^q i[\mathbf{X}_u^2,\mathbf{X}_d] + \lambda_9^q i[\mathbf{X}_u,\mathbf{X}_d^2]))^{IL}\mathbf{Y}_d^{JM}\mathbf{Y}_d^{KN}\ ,\end{aligned} \tag{3.37}$$

where $\lambda_{1,\ldots,9}^q$ are nine free complex parameters. A similar reduction can be done starting from Eq. (3.34b), leading to the alternative basis

$$\begin{aligned}(\mathbf{Y}_{udd}^{D})^{IJK} = \varepsilon^{LJK}(\mathbf{Y}_u(&\lambda_1^d\mathbf{1} + \lambda_2^d\mathbf{X}_u + \lambda_3^d\mathbf{X}_d + \lambda_4^d\mathbf{X}_u^2 + \lambda_5^d\mathbf{X}_d^2 + \lambda_6^d\{\mathbf{X}_u,\mathbf{X}_d\} \\ &+ \lambda_7^d i[\mathbf{X}_u,\mathbf{X}_d] + \lambda_8^d i[\mathbf{X}_u^2,\mathbf{X}_d] + \lambda_9^d i[\mathbf{X}_u,\mathbf{X}_d^2])\mathbf{Y}_d^{\dagger})^{IL}\ .\end{aligned} \tag{3.38}$$

[5] The soft-breaking term $\mathbf{A}_{udd}$ transforms exactly like $\mathbf{Y}_{udd}$ under the $SU(3)^3$ symmetry, so admits the same expansions, up to different expansion coefficients, and will not be detailed here.

Finally, for the last structure, Eq. (3.34c), all octet insertions but those involving $\mathbf{Y}_d\mathbf{X}_d\mathbf{Y}_u^\dagger$ and $\mathbf{Y}_d\mathbf{X}_d^2\mathbf{Y}_u^\dagger$ can be moved to the first index, and we remain with 12 possible terms. This time, there seems to be some latitude in the identification of the basis. For reasons of stability of the basis [28], the best choice is to keep two such $\mathbf{X}_d$ insertions (which have to be antisymmetrized under $J \leftrightarrow K$):

$$\begin{aligned}(\mathbf{Y}_{udd}^U)^{IJK} &= \varepsilon^{LMN}(\lambda_1^u\mathbf{1} + \mathbf{Y}_u(\lambda_2^u\mathbf{1} + \lambda_4^u\mathbf{X}_u + \lambda_5^u\mathbf{X}_d + \lambda_7^u\mathbf{X}_d^2)\mathbf{Y}_u^\dagger)^{IL}\,(\mathbf{Y}_d\mathbf{Y}_u^\dagger)^{JM}\,(\mathbf{Y}_d\mathbf{Y}_u^\dagger)^{KN} \\ &+ \varepsilon^{LMN}(\mathbf{Y}_u(\lambda_8^u\{\mathbf{X}_u,\mathbf{X}_d\} + \lambda_9^u i[\mathbf{X}_u,\mathbf{X}_d])\mathbf{Y}_u^\dagger)^{IL}\,(\mathbf{Y}_d\mathbf{Y}_u^\dagger)^{JM}\,(\mathbf{Y}_d\mathbf{Y}_u^\dagger)^{KN} \\ &+ \varepsilon^{IMN}\lambda_3^u((\mathbf{Y}_d\mathbf{X}_d\mathbf{Y}_u^\dagger)^{JM}(\mathbf{Y}_d\mathbf{Y}_u^\dagger)^{KN} + (\mathbf{Y}_d\mathbf{Y}_u^\dagger)^{JM}(\mathbf{Y}_d\mathbf{X}_d\mathbf{Y}_u^\dagger)^{KN}) \\ &+ \varepsilon^{IMN}\lambda_6^u(\mathbf{Y}_d\mathbf{X}_d\mathbf{Y}_u^\dagger)^{JM}(\mathbf{Y}_d\mathbf{X}_d\mathbf{Y}_u^\dagger)^{KN}\quad ,\end{aligned} \tag{3.39}$$

where the coefficients are ordered according to the number of Yukawa spurions.

At this stage, one may wonder why three different bases, Eqs. (3.37), (3.38), and (3.39), are constructed to parametrize $\mathbf{Y}_{udd}$. Indeed, any one of them is sufficient to project a completely arbitrary set of $\mathbf{Y}_{udd}^{IJK}$ couplings. Generalizing, it is clear that there is an infinity of equally valid bases of nine terms, at least from a mathematical point of view. Though this is indeed true when these bases are just meant to parametrize generic couplings, the situation changes when MFV is enforced. Indeed, in Ref. [28], it was shown that the MFV limit is stable and well-defined provided only one $U(1)$ is broken at a time. There are several reasons for this:

1. A first reason is that what is MFV for one basis is not necessarily MFV for another basis. To see this, consider the identity:

$$\varepsilon^{LMN}(\mathbf{Y}_u\mathbf{Y}_d^\dagger\mathbf{Y}_d)^{IL}\mathbf{Y}_d^{JM}\mathbf{Y}_d^{KN} = \det(\mathbf{Y}_d)\varepsilon^{LJK}(\mathbf{Y}_u\mathbf{Y}_d^\dagger)^{IL}\ . \tag{3.40}$$

It shows that projecting the $\lambda_1^d$ structure of the $\mathbf{Y}_{udd}^D$ basis on the $\mathbf{Y}_{udd}^Q$ basis just produces the $\lambda_3^q$ term, but that $\lambda_3^q = \lambda_1^d/\det(\mathbf{Y}_d)$. With $\det(\mathbf{Y}_d) \approx 10^{-10}\tan^3\beta$, it is clear that both $\lambda_1^d$ and $\lambda_3^q$ cannot be simultaneously of $\mathcal{O}(1)$.

2. A second reason stems from the compatibility with MFV for the $\mathcal{R}$-parity conserving soft-breaking terms, Eq. (2.26), for which the invariance under $U(3)^3$ was enforced. If the invariance under $SU(3)^3$ was imposed instead, additional terms like for example

$$(\mathbf{m}_D^2)^{IJ}/m_0^2 \ni \varepsilon^{LMN}\mathbf{Y}_u^{AL}\mathbf{Y}_d^{IM}\mathbf{Y}_d^{KN} \times \varepsilon^{RJK}(\mathbf{Y}_d\mathbf{Y}_u^\dagger)^{RA}\ , \tag{3.41}$$

should occur in their expansions, where $\varepsilon^{LMN}$ breaks $U(1)_Q$ and $\varepsilon^{RJK}$ breaks $U(1)_D$. Such structures would typically arise from the RG evolution if $\mathbf{Y}_{udd}$ contains terms breaking different $U(1)'$s. The problem is that this term does not match those already present in the expansion Eq. (2.26), and even worse, if projected onto the MFV basis of Eq. (2.26), it generates large non-MFV coefficients.

3. Finally, a last reason is related to the $U(1)$ rephasing required to maintain real and positive mass terms. Clearly, the $\mathcal{R}$-parity violating terms are sensitive to these $U(1)$ rotations, with $\mathbf{Y}_{udd}^{Q,U,D} \to \exp(iN_f\alpha_{Q,U,D})\mathbf{Y}_{udd}^{Q,U,D}$. But, provided only one $U(1)$ is broken, there remains a one-parameter freedom remaining after the shift of Eq. (3.15) allowing to get rid of that unknown phase. This cannot be done if more than one flavor $U(1)$ is broken[6].

In conclusion, only one $U(1)$ can be broken at a time for MFV to be well-defined. There are thus three possible scenarios: either $U(1)_Q$, $U(1)_U$, or $U(1)_D$ is broken. The corresponding patterns of hierarchies for the $\Delta\mathcal{B}$ couplings are shown in Table 3.2.

[6] When neutrino Majorana masses are introduced, this one parameter freedom cannot be invoked to remove the phase of $\mathbf{Y}_{udd}$ because it is conventionally used to get rid of one of the three Majorana phases present in the neutrino mass term.

| $\tan\beta$ | | Broken $U(1)_Q$ | | | Broken $U(1)_D$ | | | Broken $U(1)_U$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $ds$ | $sb$ | $db$ | $ds$ | $sb$ | $db$ | $ds$ | $sb$ | $db$ |
| 10 | $u$ | $10^{-14}$ | $10^{-9}$ | $10^{-11}$ | $10^{-9}$ | $10^{-9}$ | $10^{-9}$ | $10^{-12}$ | $10^{-6}$ | $10^{-8}$ |
| | $c$ | $10^{-10}$ | $10^{-7}$ | $10^{-7}$ | $10^{-5}$ | $10^{-7}$ | $10^{-5}$ | $10^{-13}$ | $10^{-9}$ | $10^{-10}$ |
| | $t$ | $10^{-7}$ | $10^{-6}$ | $10^{-6}$ | 0.1 | $10^{-6}$ | $10^{-4}$ | $10^{-14}$ | $10^{-13}$ | $10^{-14}$ |
| 50 | $u$ | $10^{-13}$ | $10^{-7}$ | $10^{-9}$ | $10^{-8}$ | $10^{-9}$ | $10^{-8}$ | $10^{-10}$ | $10^{-5}$ | $10^{-7}$ |
| | $c$ | $10^{-9}$ | $10^{-5}$ | $10^{-6}$ | $10^{-4}$ | $10^{-6}$ | $10^{-5}$ | $10^{-12}$ | $10^{-8}$ | $10^{-9}$ |
| | $t$ | $10^{-5}$ | $10^{-4}$ | $10^{-5}$ | 1 | $10^{-5}$ | $10^{-3}$ | $10^{-13}$ | $10^{-12}$ | $10^{-12}$ |

Table 3.2: Typical hierarchies for the modulus of the $\mathbf{Y}_{udd}$ couplings (in the superCKM basis). Because $\mathbf{Y}_{udd}^{IJK}$ is antisymmetric under $J \leftrightarrow K$, its entries can be put in a $3\times 3$ matrix form with $I = u, c, t$ and $JK = ds, sb, db$.

### Renormalization group evolution

A priori, the flavor dynamics at the origin of the MFV prescription could be active at a very high scale. It is therefore compulsory to check whether the $\mathcal{O}(1)$ assumption for the coefficient is stable throughout the evolution down to the phenomenologically relevant TeV scale. In the present section, the behavior under the renormalization group of the MFV hypothesis is briefly described.

Let us start with the broken $U(1)_Q$ scenario. It corresponds to the holomorphic restriction of MFV proposed in Ref. [93]. The hypothesis is that the flavor symmetry is dynamical at some scale $M_{\text{Flavor}}$. There, the Yukawa spurions would either be true dynamical fields, or they would be directly related to those of this unknown flavor dynamics. At the same time, supersymmetry requires the superpotential to be holomorphic, so $\mathbf{Y}_{udd}$ must be insensitive to $\mathbf{Y}_u^\dagger$ and $\mathbf{Y}_d^\dagger$ above the scale $M_{\text{Flavor}}$. The most general flavor-symmetric expansion is then very simple, since there is only one way to write $\mathbf{Y}_{udd}$ in terms of $\mathbf{Y}_u$ and $\mathbf{Y}_d$:

$$\mathbf{Y}_{udd}^{IJK} = \lambda\,\varepsilon^{LMN}\mathbf{Y}_u^{IL}\mathbf{Y}_d^{JM}\mathbf{Y}_d^{KN}\ . \tag{3.42}$$

The scale $M_{\text{Flavor}}$ at which holomorphy is imposed could be very high, and whether holomorphy survives through the evolution to the low scale is not obvious. Indeed, the RG equations of the Yukawa and $\mathbf{Y}_{udd}$ couplings are coupled (we follow the notations of Ref. [94], but for a slight change of conventions in the indices):

$$\frac{d}{dt}\mathbf{Y}_x^{IJ} = \mathbf{Y}_x^{KJ}\gamma_{X^K}^{X^I} + \mathbf{Y}_x^{IJ}\gamma_{H_x}^{H_x} + \mathbf{Y}_x^{IK}\gamma_{Q^K}^{Q^J}\ ,\ \ \frac{d}{dt}\mathbf{Y}_{udd}^{IJK} = \mathbf{Y}_{udd}^{IJL}\gamma_{D^L}^{D^K} + \mathbf{Y}_{udd}^{ILK}\gamma_{D^L}^{D^J} + \mathbf{Y}_{udd}^{LJK}\gamma_{U^L}^{U^I}\ , \tag{3.43}$$

where $x = u, d$ and $t = \log Q^2$. At one loop, $\gamma_{U^J}^{U^I}$, $\gamma_{D^J}^{D^I}$, and $\gamma_{Q^J}^{Q^I}$ all involve "non-holomorphic" spurion insertions. For example, $\gamma_{Q^J}^{Q^I}$ contains $\mathbf{Y}_u^\dagger\mathbf{Y}_u$ and $\mathbf{Y}_d^\dagger\mathbf{Y}_d$ terms. The consequence for the soft-breaking terms is well-known: even starting from universal squark masses $\mathbf{m}_Q^2 = \mathbf{m}_U^2 = \mathbf{m}_D^2 = m_0^2\mathbf{1}$ at the unification scale, the whole series of coefficients in Eq. (2.26) end up non-zero at the low scale [21, 70]. One would expect the same to happen for the $\mathbf{Y}_{udd}$ coupling: the whole series of coefficients in Eq. (3.37) would appear at the low scale.

Interestingly, the holomorphy of $\mathbf{Y}_{udd}$ holds at all scale because all these non-holomorphic effects precisely cancel out. This can be checked analytically:

$$\frac{d}{dt}\mathbf{Y}_{udd}^{IJK} = \frac{d}{dt}(\lambda\varepsilon^{LMN}\mathbf{Y}_u^{IL}\mathbf{Y}_d^{JM}\mathbf{Y}_d^{KN}) = \mathbf{Y}_{udd}^{IJK}\left(\frac{d\ln\lambda}{dt} + \gamma_{Q^P}^{Q^P} + \gamma_{H_u}^{H_u} + 2\gamma_{H_d}^{H_d}\right) + \frac{d}{dt}\mathbf{Y}_{udd}^{IJK}\ , \tag{3.44}$$

where we have used the matrix identity of Eq. (A.33a) in the form

$$\gamma_{Q^P}^{Q^P}\varepsilon^{LMN} = \varepsilon^{PMN}\gamma_{Q^L}^{Q^P} + \varepsilon^{LPN}\gamma_{Q^M}^{Q^P} + \varepsilon^{LMP}\gamma_{Q^N}^{Q^P}\ . \tag{3.45}$$

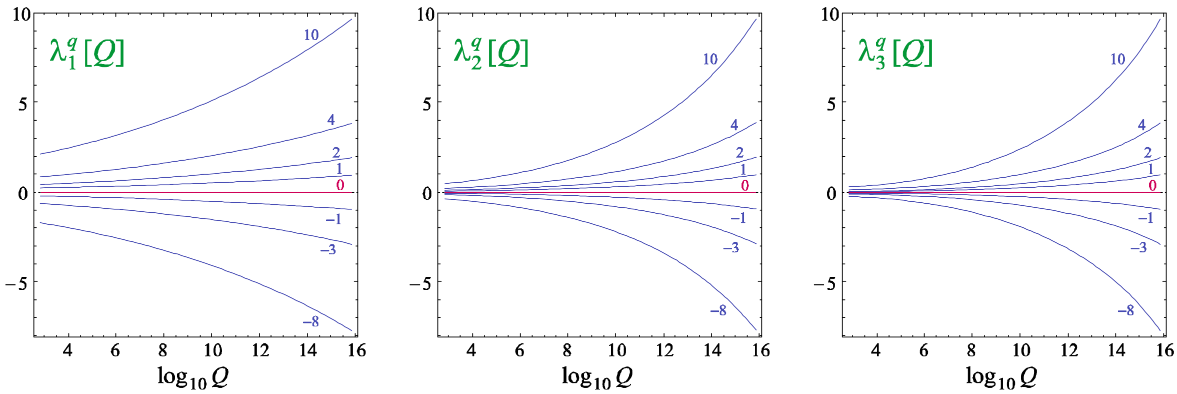


Figure 3.4: Evolution of the leading coefficients of $\mathbf{Y}_{udd}^{Q}$. The GUT scale boundary conditions are set as described in Ref. , together with $\lambda_i^q[M_{\rm GUT}] = \lambda\delta_{i1}$ (left), $\lambda_i^q[M_{\rm GUT}] = \delta_{i1} + \lambda\delta_{i2}$ (middle), and $\lambda_i^q[M_{\rm GUT}] = \delta_{i1} + \lambda\delta_{i3}$ (right), with $\lambda = \{-8, -3, -1, 0, 1, 2, 4, 10\}$. The left-hand plot corresponds to the pure holomorphic case. The other two plots show the convergence towards zero of the subleading coefficients.

Therefore, the whole evolution of the holomorphic $\mathbf{Y}_{udd}$ can be encoded into a single coefficient:

$$\frac{d\lambda}{dt} = -\lambda\beta_\lambda \;,\; \beta_\lambda = \gamma_{Q^P}^{Q^P} + \gamma_{H_u}^{H_u} + 2\gamma_{H_d}^{H_d} \;. \tag{3.46}$$

The linear dependence of $d\lambda/dt$ over $\lambda$ ensures the RG invariance of $\lambda = 0$, when $\mathcal{R}$-parity is unbroken. The beta function $\beta_\lambda$ involves only purely left-handed anomalous terms: its sole role is to compensate for the left-handed evolutions of the Yukawa couplings, since $\mathbf{Y}_{udd}$ evolves according to right-handed anomalous terms only. This explains the mechanism behind the RG invariance[7] of the $\varepsilon^{LMN}\mathbf{Y}_u^{IL}\mathbf{Y}_d^{JM}\mathbf{Y}_d^{KN}$ structure: only that term both brings in just the required combination of right-handed quark anomalous dimensions, and at the same time leaves the rest as a pure flavor trace. No other structure could be RG invariant.

At the one-loop order, $\beta_\lambda$ is given by [94]

$$\beta_\lambda^{1-\text{loop}} = \frac{1}{32\pi^2}(4\langle\mathbf{Y}_u^\dagger\mathbf{Y}_u\rangle + 7\langle\mathbf{Y}_d^\dagger\mathbf{Y}_d\rangle + 2\langle\mathbf{Y}_e^\dagger\mathbf{Y}_e\rangle - g_1^2 - 9g_2^2 - 8g_3^2) \;, \tag{3.47}$$

where $g_1$, $g_2$, and $g_3$ are the $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$ gauge couplings (with the $SU(5)$ normalization for the hypercharge). This leading order equation can easily be solved because with $\mathbf{Y}_{udd}$ entries at most of about $\lambda \times 10^{-4}$, none of the right-hand side quantities significantly depend on $\lambda$. Further, numerically, the right-hand side has only a very weak dependence on the rest of the MSSM parameters, essentially through threshold corrections. With $M_{\rm SUSY} \approx 1$ TeV, the ratio is quite stable, varying within 1/5 and 1/4 (see the first plot in Fig. 3.4).

If $\mathbf{Y}_{udd}$ is not holomorphic at some scale, it will remain so at all scales since the subleading expansion coefficients $\lambda_i^q$ of $\mathbf{Y}_{udd}^{Q}$ are non-zero. Looking back at Eq. (3.37), it is clear that these coefficients do not multiply RG invariant structures. Rather, through the evolution, each of these coefficients contributes a priori to all the others. What is remarkable is that the holomorphic scenario of Ref. [93] emerges as an infrared fixed point. Specifically, starting from some non-zero $\lambda_{i\neq1}^q$ at the GUT scale, they all evolve towards much reduced values at the low scale, with typically $\lambda_{i\neq1}^q[M_{\rm SUSY}]/\lambda_{i\neq1}^q[M_{\rm GUT}]$

[7] It is important to realize that while MFV holomorphy is an RG invariant property for $\mathbf{Y}_{udd}$, these couplings are far from invariant numerically. Not only is the coefficient evolving, but the Yukawa couplings on which $\mathbf{Y}_{udd}$ is defined are themselves scale-dependent.

of a few percents, see Fig. 3.4. To illustrate how peculiar is the behavior of $\mathbf{Y}_{udd}^{Q}$, the same analysis could be done starting with $\mathbf{Y}_{udd}^{D}$ instead (the broken $U(1)_U$ case is very similar). If the leading $\mathbf{Y}_{udd}^{D}$ structure is evolved down, i.e., if one sets $\lambda_i^d[M_{\text{GUT}}] = \delta_{i1}$, then the whole series of nine coefficients is generated at the low scale. Further, even though the behaviors of the coefficients of $\mathbf{Y}_{udd}^{D}$ remain rather smooth, and they all end up at most of $\mathcal{O}(1)$, they are not particularly small with for example $\lambda_3^d[M_{\text{SUSY}}] = 0.25$.

In conclusion, no matter the broken $U(1)$, the MFV hypothesis is perfectly compatible with the RG evolution. But, this RG point of view shed a new light on the holomorphic implementation. This is one place where the MFV hypothesis has uncovered an unexpected deeper dynamical property of the $\mathcal{R}$-parity violating coupling. Holomorphy is not only stable under the evolution, it also acts as a powerful infrared attractor. Phenomenologically, low-scale holomorphy thus systematically emerges once the broken flavor $U(1)$ is that of the quark doublet. Intriguingly, this same flavored $U(1)$ is also broken by the $\mathcal{B}+\mathcal{L}$ anomaly of the weak interactions, i.e., the $L^3Q^9$ term of Eq. (3.19) which breaks $U(1)_L$ and $U(1)_Q$. In other words, MFV, which must obviously hold in the SM, is compatible with the $\mathcal{B}+\mathcal{L}$ anomaly only if $U(1)_Q$ is broken [27]. Though the connection appears rather coincidental at present, it is thus tempting to conclude that low-scale holomorphy should hold, at least to a good approximation.

### 3.3.3 Supersymmetry search strategy: A reappraisal

It is time to recapitulate. In the previous chapter, it was shown that the MSSM suffers from the flavor puzzles. That is, its flavor couplings cannot be generic because some of them need to be tiny to pass the FCNC constraints. Actually, since no NP signal has been seen, these FCNC constraints scale like the SM contributions to these processes. So, phenomenologically, the MSSM flavored soft-breaking terms should be aligned with the flavor couplings of the SM, at least to a good approximation. In this context, the MFV hypothesis permits to achieve this automatically and naturally. Further, as a parametrization tool, it greatly reduces the number of free parameters while maintaining realistic (and radiatively stable) flavor mixing in the squark sector. One interesting consequence was the possibility to split the third generation squarks from the others, making them lighter. This helps because the LHC bounds on the quark masses are particularly tight for the first two generations.

Once MFV is active, there is no reason not to apply it on the $\mathcal{R}$-parity violating sector also. Then, MFV predicts tiny $\Delta\mathcal{L}$ couplings, proportional to neutrino masses, and highly hierarchical $\Delta\mathcal{B}$ couplings, see Table 3.2. The presence of such couplings deeply alters the supersymmetric collider phenomenology, and none of the sparticle mass bounds set in the $\mathcal{R}$-parity conserving case are expected to survive. So, it is our purpose here to analyze the signatures of the MSSM supplemented with the $UDD$ coupling, under the assumptions that $\mathbf{Y}_{udd}$ follows the hierarchies shown in Table 3.2. There are a priori two options to search for supersymmetry. Either one probes the high-energy frontier at colliders to try to directly produce the sparticles, or one pushes the high-luminosity frontier to detect its imprint in low-energy observables. Let us discuss these two options.

#### High-luminosity frontier

At low energy, there are three kinds of observables potentially affected by supersymmetric $\mathbf{Y}_{udd}$ contributions:

- $\Delta\mathcal{B} = 0$ : All the FCNC currents conserve baryon number, so they depend on $\mathbf{Y}_{udd}$ quadratically. Besides, at least two different $\mathbf{Y}_{udd}$ couplings are needed to induce a flavor transition, see Fig. 3.5. Many works have analyzed the possible impact of the $\Delta\mathcal{B}$ couplings, for example in hadronic $B$ decays [95], $b \to s\gamma$ [96], $K-\bar{K}$, $D-\bar{D}$, and $B-\bar{B}$ mixing [97]. With MFV, however,

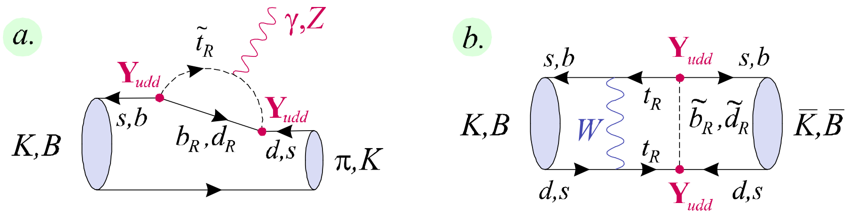


Figure 3.5: ($a$) $\mathcal{R}$-parity violating contribution to the photon and $Z$ penguins. ($b$) $\mathcal{R}$-parity violating contribution to the meson mixing boxes. Diagrams with charginos, charged Higgs, or quartic in the $\mathbf{Y}_{udd}$ couplings, are understood [97].

the virtual squark exchange is strongly suppressed. Its maximal values in the broken $U(1)_D$ scenario at $\tan\beta = 50$ are, compared to the SM contributions,

| | $b \to s$ | $b \to d$ | $s \to d$ | |
|---|---|---|---|---|
| SUSY : | $\lvert\mathbf{Y}_{udd}^{312}\mathbf{Y}_{udd}^{*331}\rvert \lesssim 10^{-3}$ | $\lvert\mathbf{Y}_{udd}^{312}\mathbf{Y}_{udd}^{*323}\rvert \lesssim 10^{-5}$ | $\lvert\mathbf{Y}_{udd}^{\dagger 331}\mathbf{Y}_{udd}^{*323}\rvert \lesssim 10^{-8}$ | (3.48) |
| SM : | $\lvert V_{ts}V_{tb}^{*}\rvert \sim 10^{-2}$ | $\lvert V_{td}V_{tb}^{*}\rvert \sim 10^{-3}$ | $\lvert V_{ts}^{*}V_{td}\rvert \sim 10^{-4}$ | |

So, these supersymmetric contributions are at best at the percent level, and thus far too small to be evidenced.

- $\Delta\mathcal{B} = 1$ : In such transitions, the number of baryons in the initial and final states differs by precisely one unit. But since baryons have spin 1/2, angular momentum conservation requires the number of leptons to also change by an odd unit, since mesons and photons have integer spin. So, these processes violate both lepton and baryon number at the same time, and the most constraining observable of this kind is by far the proton lifetime. With MFV, the current proton decay bounds are satisfied, but the predictions are not so far off the planned sensitivity of the next generation of experiments. So, at some point, proton decay is likely to be seen if MFV is to replace $\mathcal{R}$ parity. By contrast, no $\Delta\mathcal{B} = 1$ transitions should ever be seen in meson or lepton decays because the $\Delta\mathcal{L}$ couplings are way too small.

- $\Delta\mathcal{B} = 2$ : To circumvent the strong suppression brought in by the tiny lepton number violating couplings, there are several purely baryonic processes able to directly probe the $\Delta\mathcal{B} = 2$ effective interactions at low energy, which are quadratic in $\mathbf{Y}_{udd}$ (or $\mathbf{Y}_{udd}^{\dagger}$). The most constraining are neutron-antineutron oscillations and dinucleon decays like $p^+p^+ \to K^+K^+$. As for proton decay, the current bounds are satisfied but not always by a large margin, especially for squark masses below about 500 GeV. Further progress would be needed to fully exploit these observables, both experimentally and theoretically. At present, the sensitivity of the dinucleon decays appear very promising but the theoretical prediction suffers from very poorly known hadronic matrix elements [93].

Once MFV is imposed, supersymmetric effects at low energy will be difficult to see, especially if sparticle masses are close or above 1 TeV. Let us thus take the high energy route of the colliders.

**High-energy frontier**

In the $\mathcal{R}$-parity conserving case, the simplest production mechanisms for supersymmetric particles at the LHC are driven by the supersymmetrized QCD part of the MSSM. Further, processes like $d\,d \to \tilde{d}\,\tilde{d}$ or $g\,g \to \tilde{g}\,\tilde{g}$ have very large cross-section when the on-shell $\tilde{d}$ or $\tilde{g}$ production is kinematically

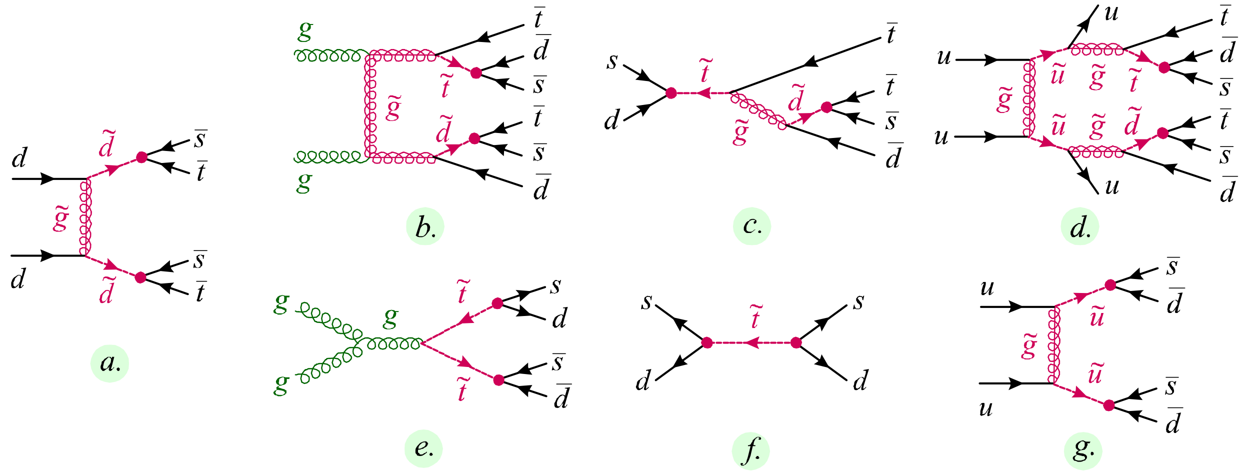


Figure 3.6: $(a-d)$ Examples of mechanisms leading to same-sign top pair final states, starting from the dominant QCD processes producing sparticles out of two colliding protons. $(e-g)$ Examples of production mechanisms leading to light-quark jet final states.

accessible, hence the tight bounds already set on these particle masses. As stressed before, these bounds assume the presence of a significant missing energy in the final state and only hold if $\mathcal{R}$ parity is conserved.

When the largest $\Delta\mathcal{B}$ coupling is smaller or comparable to $\alpha_S$, squarks and gluinos are still mostly produced in pair through QCD processes. At the LHC, the most abundantly produced sparticle states are those associated to the proton valence quarks and gluons, that is $gg \to \tilde{g}\tilde{g}$, $uu \to \tilde{u}_i\tilde{u}_j$, $dd \to \tilde{d}_i\tilde{d}_j$, etc, where $i,j = L,R$. The main non-QCD production mechanism is the single squark resonant production, for example $sd \to \tilde{t}$ with $s$ a proton sea quark, because it requires less center-of-mass energy. The main difference with the $\mathcal{R}$-parity conserving case is that once the $\mathbf{Y}_{udd}$ couplings are turned on, each of these sparticles initiates a decay chain ending with quark final states, resulting in a significant hadronic activity instead of missing energy. If we assume that the charginos and sleptons are heavier than squarks, gluinos, and the lightest neutralino (denoted simply as $\tilde{\chi}^0 \equiv \tilde{\chi}^0_1$ in the following), then we can identify three main characteristic signatures in this hadronic activity:

1. **Top-quark production** including same-sign top pairs. Because the dominant $\mathbf{Y}_{udd}^{IJK}$ couplings are those with $I = 3$, most processes lead to top quarks in the final states (see Fig. 3.6). For example, we have $\tilde{d} \to \bar{t}\,\bar{s}$ or $\tilde{g}, \tilde{\chi}^0 \to t\,d\,s,\ \bar{t}\,\bar{d}\,\bar{s}$. Even the stop can decay into top-quark pairs if $\tilde{t} \to \tilde{g}\,t$ or $\tilde{t} \to \tilde{\chi}^0\,t$ is kinematically open (see Fig. 3.6$c$). For all these modes, the production of same-sign top pairs is always possible thanks to the Majorana nature of the gluino and neutralino. In that case, the overall process is $\Delta\mathcal{B} = \pm 2$, and corresponds to a dynamical implementation of the effective operators of Eq. (3.22). Crucially, this dynamics strongly enhance the signal. When the center of mass energy is sufficient to produce the two sparticles on shell, the cross section becomes independent of the value of $\mathbf{Y}_{udd}$ since once produced, each sparticle has to decay 100% of the time. The only constraint is for $\mathbf{Y}_{udd}$ not to be too small, otherwise the sparticle would live too long, see point 3 below.

2. **Di- or trijet resonances** built over light quarks and maybe a few $b$ quarks. A priori, dijets could originate from squark decays and trijets from gluino or neutralino decays. But with MFV, only up-type intermediate squarks can lead to light-quark jets, since the other sparticle decay products always include a top quark. The simplest process is thus the $\Delta\mathcal{B} = 0$ resonant stop production with a dijet final state (see Fig. 3.6$f$). But since the electric charge of a jet is

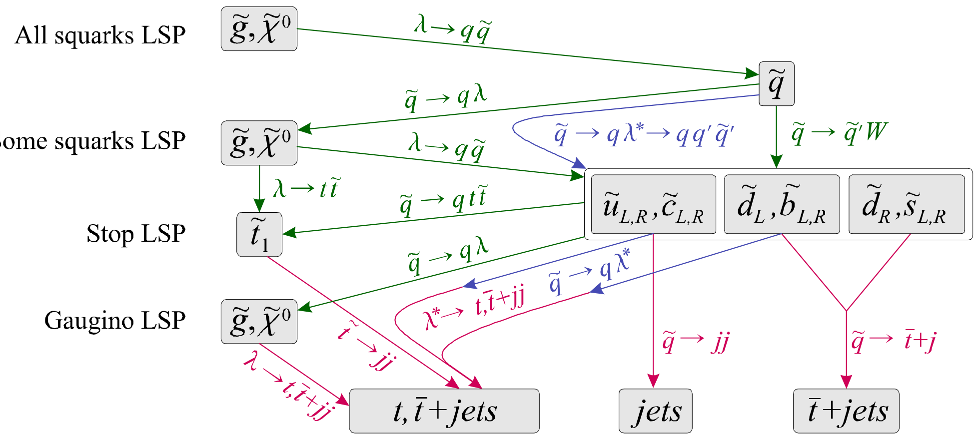


Figure 3.7: Decay chains for various MSSM mass spectra. The symbol $\lambda^{(*)}$ denotes a real (virtual) gluino or neutralino. For each squark, the relative strengths of the $\mathcal{R}$-parity conserving decay to on-shell (green) or off-shell (blue) gauginos, and $\mathcal{R}$-parity violating (red) transitions depend on the details of the mass spectrum as well as on the MFV hierarchy. When $U(1)_D$ is broken, jets are mostly made of light quarks, while with $U(1)_Q$ broken, a good fraction of them would be built upon $b$ quarks instead (see Table 3.2).

not measurable, the $\Delta\mathcal{B}$ nature of the transition cannot be ascertained and QCD backgrounds appear overwhelming. Nevertheless, given the potentially large cross sections, such an uncharacteristic enhanced jet activity could be accessible experimentally [98], and has already been searched for at colliders (see e.g. Ref. [99]).

3. **Long-lived exotic states**, the so-called $\mathcal{R}$-hadrons built as hadronized squarks or gluinos flying away [100]. Such quasi-stable exotic states have already been searched for experimentally, excluding squark masses below about 600 GeV and gluino masses below about 1 TeV [101]. With MFV, such $\mathcal{R}$-hadron signatures are rather difficult to get because some $\Delta\mathcal{B}$ couplings are large and all sparticles can find a way to use them for decaying. For example, even if the $\tau$ slepton is the LSP, it can still decay via a virtual neutralino, $\tilde{\ell} \to \ell + \tilde{\chi}^0 \to \ell + t\,d\,s$. Note though that $\mathcal{R}$-hadron signals are not impossible to get, but require large mass splitting among sparticles. For example, long-lived gluino LSP can arise if squark masses are well beyond the TeV scale, as for example in the split SUSY scenario [102], since the $\tilde{g} \to tds$ decay channel proceeds via a virtual squark. Alternatively, the neutralino could become long-lived if it is lighter than the top quark. The best handle would then be the search for the monotop signals [103] produced via $\bar{s}\,\bar{d} \to \bar{\tilde{t}} \to \bar{t}\,\tilde{\chi}^0$. Finally, it should be mentioned that top identification for point 1 above relies on that of a $b$ jet, which is possible only when the sparticle does not fly for more than a few centimeters [104].

The relative and absolute strengths of these signals depend crucially on the MSSM mass spectrum[8]. To proceed, we should characterize the different mass spectra and corresponding decay chains in details. This is a rather technical discussion whose main outcome is depicted in Fig. 3.7, which shows that most sparticle decay chains end with top quarks (see Ref. [30] for the full analysis). In view of its near-universality, let us thus concentrate on the same-sign top signature (see also Refs. [105–107]). To identify this final state, despite its relatively small 5% probability, the same-sign dileptons are best

[8]This depends also on the chosen $U(1)$ breaking. In the following, only the broken $U(1)_Q$ and $U(1)_D$ scenarios are considered. Indeed, if $U(1)_U$ is broken, then looking at Table 3.2, the (s)top couplings never exceed $\mathcal{O}(10^{-13})$, the same-sign top quark signals would be mostly replaced by the more challenging two or three light-jet resonances.

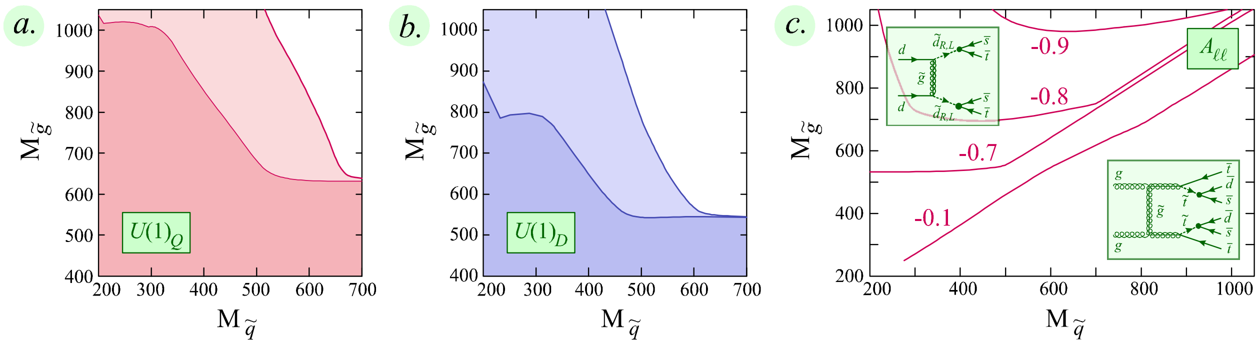


Figure 3.8: $(a,b)$ Exclusion contours at 95% CL in the $M_{\tilde{q}} - M_{\tilde{g}}$ plane derived from the CMS same-sign dilepton search [111] for the broken $U(1)_Q$ and $U(1)_D$ scenarios. The lower contours are very conservative, and obtained with the contributions from intermediate $\tilde{d}_R$ only. The upper ones are more realistic and assume an equal top production from $\tilde{d}_R$ and $\tilde{d}_L$. The presence of a light stop or a light neutralino does not significantly impact these exclusion regions. The bounds are typically tighter for the holomorphic scenario thanks to the more numerous $b$-quark jets. $(c)$ Lepton charge asymmetry of Eq. (3.49) exhibited by the same-sign dilepton signal. It gets close to $-1$ in the upper half plane, where $dd \to \tilde{d}\tilde{d}$ dominates.

suited. There are several reasons for this. First, charged leptons are clearly identified in detectors and avoid jet combinatorial background. Second, they allow to determine almost unambiguously the sign, and therefore the baryon number, of the top quarks they arose from. Finally, irreducible backgrounds are small as same-sign dilepton production is rare in the SM, arising mostly from the electroweak processes $pp \to t\bar{t}W$ [108] and $pp \to t\bar{t} + Z$ [109], with one top quark and the gauge boson decaying leptonically.

Both CMS [110,111] and ATLAS [112,113] have studied the same-sign dilepton signature at 7 and 8 TeV, and used it to set generic constraints on NP contributions. In Ref. [30], we used the former data, since the information (including efficiencies) and guidelines for constraining any model in an approximate way is provided. The exclusion regions in the gluino - squark mass plane are shown in Fig. 3.8. Note that the squark mass $M_{\tilde{q}}$ is understood as those of the first two generations. The sensitivity of the same-sign dilepton signal to the stop mass is very weak. Actually, in the extreme case of a stop LSP with very heavy first two-generation squarks (the natural SUSY scenario which, as discussed in Chapter 2, is compatible with MFV), then the relevant parameter is the mass of the gluino since the dileptons would originate from $pp \to \tilde{g}\,\tilde{g}$ with the gluino decaying as $\tilde{g} \to t\,b\,s, \bar{t}\,\bar{b}\,\bar{s}$ via on-shell $\tilde{t}$ squarks.

In the future, these exclusion regions are expected to creep upwards. Improving the limits by a factor of ten could lead to an increase of the absolute bound on the gluino mass of the order of a couple of hundred GeV. The improvement would be the more significant in the lowest allowed squark mass region where the limit on the gluino mass could increase by more than a factor of two. A similar gain would be obtained at the 14 TeV LHC if a bound on the non-standard same-sign dilepton fiducial rate comparable to the one obtained so far at 8 TeV is achieved. Pushing these limits well beyond the TeV appears difficult though. The characteristics of the signal change as the sparticles get heavier, and adequate techniques should be put in place e.g. to identify the boosted top quarks (see for instance Ref. [114]).

At the same time, current CMS and ATLAS same-sign lepton searches are not tailored for this MFV scenario. They are even not best suited to search for $\not{B}$ violation in general. Their cuts in transverse missing energy $\not{E}_T$ or hadronic activity ($H_T$, jet multiplicity, jet $p_T$, etc.) are not optimal and could be improved, and the possibility to exploit displaced vertices, if any, should be investigated [93].

There is yet another piece of information which may help a lot. The initial state at the LHC has a $\mathcal{B} = +2$ charge since it is made of two protons. It is not invariant under charge conjugation, so a significant lepton charge asymmetry defined as [29]

$$A_{\ell\ell'} \equiv \frac{\sigma(pp \to \ell^+\ell'^+ + X) - \sigma(pp \to \ell^-\ell'^- + X')}{\sigma(pp \to \ell^+\ell'^+ + X) + \sigma(pp \to \ell^-\ell'^- + X')} , \tag{3.49}$$

can be expected. For instance, the SM same-sign dilepton production features a predominance of positively charged dileptons over negative ones of about 25% because $u\bar{d} \to \bar{t}tW^+$ happens more often than $\bar{u}d \to \bar{t}tW^-$. On the other hand, the $\Delta\mathcal{B}$ processes initiated by down valence quarks (that dominate the same-sign dilepton production when squarks are lighter than gluinos) are significantly more probable than their conjugates, initiated by anti-down quarks. In the upper-left part of the $M_{\tilde{q}} - M_{\tilde{g}}$ plane, much more anti-top than top-quark pairs are therefore expected. This leads to a predominance of negatively charged dileptons and $A_{\ell\ell'}$ approaches $-1$ for all $\ell, \ell' = e, \mu, \tau$ (see Fig. 3.8, where only electrons and muons are considered).

This observation has two important consequences. On the theoretical side, as emphasized in Ref. [29], such a negative asymmetry is a smoking gun for NP and an important evidence for baryon number violation. It is indeed almost impossible to obtain in other realistic NP scenarios. On the experimental side, a precise measurement of this asymmetry, in which systematic uncertainties cancel, could provide better constraints. In addition, a limit on the production rate of negatively charged lepton pairs only, for which SM irreducible backgrounds are smaller, could in principle be used to improve the current bounds in the upper half of the $M_{\tilde{q}} - M_{\tilde{g}}$ plane.

In conclusion, though baryonic $\mathcal{R}$-parity violation may appear as a naughty twist of Nature, requiring us to delve into the intense hadronic activity of proton colliders, the LHC may actually be well up to the challenge. First, most of this hadronic activity should be accompanied with top or anti-top quarks, which can be efficiently identified by both CMS and ATLAS. Second, from a baryon number point-of-view, the LHC is an asymmetric machine since it collides protons. This could prove invaluable to disentangle $\Delta\mathcal{B}$ effects from large SM backgrounds. So, $\mathcal{R}$-parity violating low-scale supersymmetry should not remain unnoticed for long under the onslaught of the future nominal 14 TeV collisions.

# Outlook and conclusion

In this report, we have presented the MFV hypothesis. All starts from the peculiar SM flavor sector, and the correspondingly peculiar flavor and $\mathcal{CP}$-violating phenomenology. Thanks to the many severely suppressed or even forbidden observables, very tight constraints are drawn on possible NP contributions. Taken at face value, these bounds seem to rule out even the mere presence of new flavored particles or new interactions among quarks or leptons at the TeV scale. This is problematic theoretically, because we need some new dynamics there to make sense of the SM Higgs mechanism. The purpose of MFV is to circumvent this so-called flavor puzzle by naturally suppressing the flavor breaking effects induced by any new dynamics at the TeV scale.

In practice, MFV can also be viewed as an instrument in our quest for a NP signal. Indeed, the complementarity between direct searches at colliders and indirect searches at flavor factories is essential. Naively, the former can measure the masses of new particles, while the latter can constrain their couplings. Of course, these two types of measurements only make sense when taken together, since in general, the strengths of the processes at colliders also depend on the couplings, and the indirect low-energy effects on the new particle masses. To make this connection, MFV offers the perfect tool. For instance, if a NP signal emerges from flavor physics, whether in $K$ or $B$ decays, lepton flavor violating processes, lepton or baryon EDM,..., then MFV permits to characterize the flavor structures involved, and thereby helps in the direct identification of the new particles at the LHC, for example by guiding us towards the most promising discovery channels.

Going forwards, what the future holds for the SM is difficult to guess. Particle physics is an experimental science, and right now it happens to sit right at a crossroad. Current experiments, essentially the LHC but also flavor factories, are entering completely unexplored realms. In the very near future, maybe in the next few weeks or even days, something could be discovered. Though nobody can tell what it could be (supersymmetry, one or more Higgs bosons, new dimensions, forces, or forms of matter,. . . ), such a discovery would most probably define the whole of particle physics for decades.

In this context, the MFV approach offers a framework that will survive no matter what unravels experimentally. Indeed, in this report we deliberately choose to ground MFV on a rather universal symmetry formalism. It will always be a valid tool to explore the flavor sector of any NP model, maybe at the cost of introducing one or more spurions in addition to the Yukawa couplings. Furthermore, current results from flavor factories prove that there cannot be so many new flavor structures, otherwise NP would have been discovered by now. So, both phenomenologically or as a tool, MFV cannot fail and remains useful in the LHC era.

Besides this general statement, and even if it is a bit tricky in our physically uncertain times, there are a number of specific topics related to MFV which could be worth investigating in the near future. So to conclude this report with concrete perspectives beyond what was presented, let us briefly review promising subjects, categorizing them into three areas of research: phenomenology, model-building, and cosmological implications.

### MFV phenomenology

- **Leptonic sector**: Due to lack of space, we did not discuss at all the MFV implementation in the leptonic sector, first introduced in Ref. [115], and this even though the literature is quite extensive (see e.g. Ref. [23, 116]). Typically, the goal of these analyses is to predict the possible NP enhancement of lepton flavor violating transitions like $\mu \to e\gamma$, making use of neutrino oscillation data. If neutrinos were Dirac particles, this would trivially parallel the treatment presented here for the quark sector. But, being so light, naturality dictates some alternative mechanism to generate their masses, e.g. with a seesaw mechanism. In that case, additional leptonic spurions are present, like a $\Delta\mathcal{L} = 2$ Majorana mass term, whose background values cannot be entirely fixed. For example, in the minimal seesaw [1], 18 new parameters are introduced (19 if $\mathcal{B}$ is separately violated), but neutrino oscillations only give access to two mass differences and three mixing angles. This renders the predictions for LFV or $\mathcal{CP}$-violation in the lepton sector far more challenging.

- **Flavored benchmarks for New Physics**: Often, the initial settings and benchmarks used to probe for new dynamics assume an oversimplified flavor sector. For instance, to identify the particles of the MSSM, many simulations use flavor-blind sparticle couplings. However, as explained in Chapter 2, this is not consistent with the presence of flavor violation already at the level of the SM. In this context, to account completely for all the richness of a non-trivial flavor sector, all the while keeping the low-energy experimental constraints in check, MFV offers an ideal setting. A preliminary study along that line was done in Ref. [73] for the MSSM, and much work remains within that model as well as for alternative scenarios. In particular, the full implementation of the MFV formalism in the various computer tools and spectrum calculators should be undertaken.

- **$\mathcal{B}$ violation at the LHC**: One of our main results is to have proven the compatibility of $\mathcal{B}$ violation with TeV-scale NP. Though a proton collider like the LHC is not the best equipped to deal with that kind of physics, given the expected intense hadronic activity, the sensitivity may nevertheless be sufficient to get a compelling signal. In this context, optimizing the search strategies could prove very fruitful, both within the MSSM [30] or model-independently [29]. This includes for example displaced vertices [93], dilepton charge asymmetries [29], and maybe top polarizations [117].

### MFV model-building

- **TeV-scale extensions to the SM**: When constructing extensions of the SM at the TeV scale, flavor constraints immediately kick in. Instead of going through the delicate process of identifying and then fine-tuning each of the dangerous parameters of the model, simply enforcing MFV immediately improves its viability. This strategy has already been applied to study the low-energy and collider signatures of many flavored models besides supersymmetry [118], e.g., multi-Higgs models [119], vector-like quarks [121], colored scalar resonances [120], extra dimensions [122], leptoquark interactions [123],... and could certainly be pursued further.

- **Grand Unified Theories**: There are two pressing issues in this context. First, the formulation of MFV itself, which requires identifying a minimal set of spurions, is not entirely settled. Indeed, this is less easy that it seems for GUT like $SU(5)$ or $SO(10)$ because their minimal flavor content is not compatible with the observed fermion masses. For example, minimal $SU(5)$ predicts $m_s/m_d \approx m_\mu/m_e$, which is badly violated. So, their flavor sector must be amended, bringing in additional spurions whose background values are not entirely known. In the first attempt [125] at applying the MFV idea to $SU(5)$, this prevented a complete control over the flavor transitions, and some flavor mixing had to be cancelled by hand. This should be further investigated [124]. A second issue is the incompatibility of a conserved $\mathcal{B}$ or $\mathcal{L}$ with the unification of quarks and leptons. There are two types of $\mathcal{B}$-violating interactions. Those arising from the exchange of very heavy gauge bosons are flavor-blind and beyond the reach of MFV. The only way to control them is to push the GUT-breaking scale sufficiently high. The second type is flavored, and arises when the GUT is supersymmetrized. Naively, the $\mathcal{R}$-parity violating GUT couplings induce both $\mathcal{B}$ and $\mathcal{L}$ violation at the low scale, even without neutrino masses, and proton decay comes back haunting us. This is not automatic though. Models where only the MFV-like $\mathcal{B}$ violating coupling arises have already been constructed [126], but a systematic study of when and how this happens has not been performed yet.

- **Origin of MFV**: In Chapter 2, it was shown by analogy with low-energy QCD that the mere existence of MFV relations may well leave us without any clue about their origin. Still, there is no question that if MFV holds, it must come from somewhere. So, constructing models in which MFV arises dynamically is an important topic, from which some hints about the high-energy flavor dynamics may be gleaned. In this context, many works tackle either the $\mathcal{B}$ conserving or $\mathcal{B}$ violating sector, but not both at the same time [64]. Further, these analyses do not always try to relate the origin of MFV to that of the known flavor structures (the SM Yukawa couplings). Of course, a comprehensive theory of flavor is still a long way off, but since ultimately this is one of the main goals of modern particle physics, it should certainly stay on our radar.

**MFV cosmology**

- **Dark matter**: There are two classes of topics related to MFV. The first introduces the idea of a flavored dark matter particle (DM), in the sense that its coupling to matter fermions would not be universal [127]. This is often welcome to reconcile the relatively low DM relic density observed with the results of the current direct detection experiments. Indeed, the former asks for significant annihilation/decay channels towards normal matter, hence significant DM couplings to SM particles, in contradiction with the latter unsuccessful searches. The loophole here is that these experiments attempt to collide wandering DM particles with large amounts of nuclei, so they are only sensitive to the couplings to light quarks making up the nucleons. With MFV, hierarchical couplings of the DM with quarks would naturally suppress those with the first generations. On the other hand, the coupling to top quarks may remain relatively large, opening the way for associated top + DM productions at the LHC.

  As a special case, the second class of works tackles the very stability of the DM candidate by adapting the MFV strategy we proposed for the proton [128], thereby avoiding the need for some ad-hoc discrete symmetries. After all, we could insure a lifetime greater than $10^{33}$ years, which is more than enough from a cosmological point of view. Here also, DM couplings to heavy third generation fermions are typically favored, exactly like for the $\Delta\mathcal{B}$ couplings studied in Chapter 3, and could lead to interesting signatures at the LHC.

  These two topics are still very active areas of research, and further works along these lines are envisioned.

- **Axions**: In Chapter 3, we discussed the $U(1)$ flavor symmetries and identified those corresponding to $\mathcal{B}$, $\mathcal{L}$ and the $\mathcal{PQ}$ symmetry. Axions are associated to the breaking of this latter $U(1)$. But, looking back at Eq. (3.8), it is clear that these $U(1)$s are not orthogonal. In general, $\Delta\mathcal{B}$ couplings carry non-trivial $\mathcal{PQ}$ charges. So, if $\mathcal{B}$ is violated, one should expect $\mathcal{B}$ violating processes involving the axion. Constructing a viable model is a bit tricky though, because the $\mathcal{PQ}$ charges of generic $\mathcal{B}$ violating couplings are fractional. Anyway, here is a connection between two apparently unrelated fundamental questions of the SM that should be further investigated [129], especially in view of the MFV implications for DM searches mentioned previously.

- **Baryon Asymmetry of the Universe**: A final topic in which MFV could play a role is the identification of the mechanism having led to the current matter-antimatter asymmetry of the Universe. Once again, there are two different aspects. First, as discussed previously, applying MFV to the leptonic sector is rendered tricky by the many free parameters in the neutrino spurions. To at least partially constrain those, one possibility is to require a successful leptogenesis [81]. In that case, the very heavy right-handed neutrinos would bear the responsibility of generating an $\mathcal{L}$ asymmetry through their cascade decays, tuned by MFV [130]. In a second time, the SM $U(1)_{\mathcal{B}+\mathcal{L}}$ anomaly would convert this asymmetry into the observed $\mathcal{B}$ asymmetry.

  The leptogenesis scenario has been extensively studied, but there is an alternative within MFV. As we discussed, large $\mathcal{B}$ violating couplings can be expected. A priori, those could thus directly induce the asymmetry of the Universe, and there is no need to go through $\Delta\mathcal{L} = 2$ effects first. In fact, if $\Delta\mathcal{B}$ couplings are active at the TeV scale, they should better play a role for the asymmetry because otherwise, they could well wipe out any pre-existing baryonic imbalance. For this to work though, the other Sakharov conditions, $\mathcal{CP}$ violation and out-of-thermal equilibrium transitions, must be fulfilled. Though there have been some works along this line recently [131], this idea still needs to be further explored. Also, any connection with flavored axion and/or flavored DM models is still totally unexplored.

In conclusion, MFV is no longer just the convenient phenomenological tool it was initially meant to be. Nowadays, as this list of topics shows, its scope of application is impressively broad, ranging from low-energy physics to cosmology. Not only does MFV offer a gateway towards many of the most promising theoretical ideas and models, it may even help resolve some of the most puzzling fundamental questions left open by the Standard Model.

# Appendix A

# Supplementary material

## A.1 Bounds on the FCNC operators

In this Appendix, we briefly describe the extractions of the various bounds on the scale $\Lambda$ discussed in Chapter 2 from the experimental data on flavor observables.

### Meson mixing and box diagrams

Evaluating the contribution of the $\Delta S = 2$ vector operator $(\bar{s}_L\gamma^\mu d_L)^2$ to $\varepsilon_K$, and requiring that it does not exceed the experimental value, we find

$$|\varepsilon_K|^{NP} = \frac{\mathrm{Im}\langle K^0|\mathcal{H}_{eff}^{NP}|\bar{K}^0\rangle}{\sqrt{2}\Delta M_K} = \frac{\mathrm{Im}\,\mathcal{C}_{WW}^{21}}{\Lambda^2}\frac{4B_K F_K^2 m_K}{3\sqrt{2}\Delta M_K} \lesssim |\varepsilon_K|^{\mathrm{exp}} , \tag{A.1}$$

where the relevant hadronic matrix element is $\langle K^0|\,(\bar{s}_L\gamma^\mu d_L)\,(\bar{s}_L\gamma_\mu d_L)\,|\bar{K}^0\rangle \equiv 4/3 B_K F_K^2 m_K$ with $B_K \approx 3/4$ [132], $F_K \approx 113\,\mathrm{MeV}$, while $\Delta M_K^{\mathrm{exp}} = 3.483(6)\times 10^{-15}$ GeV and $|\varepsilon_K|^{\mathrm{exp}} = (2.232 \pm 0.007)\times 10^{-3}$ [17]. To derive the scales quoted in the main text, we neglect the interference between the SM and NP contribution. It is interesting to compare these scales to those one would obtain for a scalar operator like $(\bar{s}_R d_L)\,(\bar{s}_L d_R)$. Because of its relationship with quark masses, there is a chiral enhancement of the matrix element by

$$\langle K^0|\,(\bar{s}_R d_L)\,(\bar{s}_L d_R)\,|\bar{K}^0\rangle \sim \left(\frac{m_K^2}{m_s + m_d}\right)^2 F_K^2 m_K B_K^{LR} , \tag{A.2}$$

where $B_K^{LR}$ is of $\mathcal{O}(1)$, together with a corresponding strong enhancement of the Wilson coefficients through its QCD evolution down to the hadronic scale. At the end of the day, the constraint on the scale $\Lambda$ for such an operator are an order of magnitude above those for the $(\bar{s}_L\gamma^\mu d_L)^2$ operator.

The $B_q^0 - \bar{B}_q^0$ mixing master equations are [37]

$$\Delta M_q = \frac{1}{M_{B_q}}|\langle\bar{B}_q^0|\mathcal{H}_{eff}^{\Delta B=2}|B_q^0\rangle| \ , \ \mathcal{H}_{eff}^{\Delta B=2} = \frac{1}{M_W^2}[\mathcal{C}_{WW}^{3q}]_{\mathrm{SM}} Q_q^{\Delta B=2} ,$$
$$[\mathcal{C}_{WW}^{3q}]_{\mathrm{SM}} = \frac{g^4}{32\pi^2}(V_{tb}^\dagger V_{tq})^2 \eta_B S_0(x_t) \ , \ \langle B_q|Q^{\Delta B=2}|\bar{B}_q\rangle = \frac{2}{3}B_{B_q}F_{B_q}^2 M_{B_q}^2 , \tag{A.3}$$

where $Q_q^{\Delta B=2} = (\bar{b}_L\gamma_\mu q_L)^2$, $\eta_B \approx 1/2$ is a strong correction, $\sqrt{B_{B_d}}F_{B_d} \approx 200$ MeV and $\sqrt{B_{B_s}}F_{B_s} \approx 240$ MeV from lattice calculations [132], while finally $S_0(x_t = m_t^2/M_W^2) \approx 2.46$ is the value of the short-distance box diagram. Altogether, this thus gives

$$\Delta M_q^{\mathrm{SM}} = \frac{2B_{B_q}F_{B_q}^2 M_{B_q}}{3M_W^2}|\mathcal{C}_{WW}^{3q}|_{\mathrm{SM}} = \frac{G_F^2}{6\pi^2}\eta_B M_{B_q} B_{B_q} F_{B_q}^2 M_W^2 S_0(x_t)(V_{tb}^\dagger V_{tq})^2 . \tag{A.4}$$

The measured values are $\Delta M_d^{\exp} = 0.510 \pm 0.003\, ps^{-1}$ and $\Delta M_s^{\exp} = 17.761 \pm 0.022\, ps^{-1}$ [17], which permit to tightly constrain $V_{td}$ and $V_{ts}$, see Fig. 2.1. The bounds in the text are then obtained by requiring that a NP contribution does not exceed the measured value, that is

$$\frac{2B_{B_q} F_{B_q}^2 M_{B_q}}{3\Lambda_q^2} |\mathcal{C}_{WW}^{3q}|_{\rm NP} \lesssim \Delta M_q^{\exp} \,, \tag{A.5}$$

with various assumptions on $|\mathcal{C}_{WW}^{3q}|_{\rm NP}$. These scales do not change by much if we require instead the NP contribution not to exceed 10% say of $\Delta M_q^{\exp}$. In any case, we cannot fully benefit from the excellent experimental precision because of the theory error on the SM contribution.

### Semileptonic decays and $Z$ boson penguins

From the effective $Z$ boson operators, coupling the $Z$ boson to a lepton pair produces effective four-fermion operators [37]

$$\mathcal{H}_{eff}^{NP} = \frac{\mathcal{C}_Z^{IJ}}{\Lambda^2} (\bar{q}^I \gamma_\mu P_L q^J)(\bar{\nu}^K \gamma^\mu P_L \nu^K + \bar{\ell}^K \gamma^\mu (8 \sin^2\theta_W - P_L)\ell^K) + h.c. \,. \tag{A.6}$$

In the kaon sector, the best constraints come from the very rare $K^+ \to \pi^+ \nu\bar{\nu}$ process. Neglecting the SM contribution and imposing that it is not overproduced gives

$$\frac{m_{K^+}^5}{512\pi^2} \frac{\Phi^+}{\Gamma_+} \frac{|\mathcal{C}_Z^{21}|_{\rm NP}^2}{\Lambda^4} \lesssim \mathcal{B}\left(K^+ \to \pi^+ \nu\bar{\nu}\right)^{\exp} = (14.7^{+13.0}_{-8.9}) \times 10^{-11} \;\; [133] \,, \tag{A.7}$$

where $\Phi^+ \approx 0.14$ is a phase-space integral, and $\Gamma_+ = 5.3 \times 10^{-14}$ MeV is the $K^+$ width [17]. In the $B$ sector, the tightest bounds come from the $B_{s,d} \to \mu^+\mu^-$ modes (for which the $\gamma$ penguin does not contribute), with neglecting again the interference with the SM,

$$\mathcal{B}(B_q^0 \to \ell^+\ell^-)_{\rm NP} = \tau_{B_q} \frac{M_{B_q} F_{B_q}^2 m_\ell^2}{32\pi\Lambda^4} \sqrt{1 - 4m_\ell^2/M_{B_q}^2} |\mathcal{C}_Z^{3q}|_{\rm NP}^2 \,, \tag{A.8}$$

where $\langle 0|\bar{b}\gamma_\mu\gamma_5 q|B_q(p_B)\rangle = iF_{B_q} p_B^\mu$ with the lattice estimates [132] $F_{B_s} \approx 0.235$ GeV and $F_{B_d} \approx 0.195$ GeV. In the SM [37], $\mathcal{C}_Z^{21} = \sqrt{2} G_F \alpha/(\pi \sin^2\theta_W) V_{tb}^\dagger V_{tq} Y(m_t/M_W)$ with $Y(m_t/M_W)$ the Inami-Lim function for the $W$ boson-top quark loop (it is not exactly the same function as for the neutrino modes because of the presence of semileptonic $W$ boxes, ensuring a gauge invariant result). The bounds in the text are obtained by requiring $\mathcal{B}(B_q^0 \to \mu^+\mu^-)^{\rm NP} \lesssim \mathcal{B}(B_q \to \mu^+\mu^-)^{\exp}$, with NP contribution not to exceed the measurements $\mathcal{B}(B_s \to \mu^+\mu^-)^{\exp} = (3.1 \pm 0.7) \times 10^{-9}$ and $\mathcal{B}(B_d \to \mu^+\mu^-)^{\exp} < 6.3 \times 10^{-10}$ [134].

### Radiative decays and magnetic penguins

For the magnetic operator, consider first $b \to s\gamma$, for which the electroweak magnetic operator is usually written as

$$\mathcal{H}_{eff}^{EW} = \frac{G_F}{\sqrt{2}} \times V_{tb}^* V_{ts} \times C_{7\gamma}^q Q_{7\gamma} \,, \; Q_{7\gamma}^q = \frac{e m_b}{4\pi^2} \times \bar{b}_R \sigma_{\mu\nu} q_L F^{\mu\nu} \,, \tag{A.9}$$

with $q = s, d$. Then, defining $\delta C_{7\gamma}^s = C_{7\gamma}^s - C_{7\gamma}^{s,SM}$, the deviation from the SM prediction induced by NP contributions is [135]

$$B(b \to s\gamma) = B(b \to s\gamma)_{SM} - (8.22 \times 10^{-4})\delta C_{7\gamma}^s + \mathcal{O}(C_{7\gamma}^{s,2}) \,. \tag{A.10}$$

With $\mathcal{B}(b \to s\gamma)^{\rm SM} = (3.36 \pm 0.23) \times 10^{-4}$ and $\mathcal{B}(b \to s\gamma)^{\rm exp} = (3.43 \pm 0.21 \pm 0.07) \times 10^{-4}$, we get $\delta C_{7\gamma}^{s} \lesssim 0.1$. The corresponding bounds on $\delta C_{7\gamma}^{d} \lesssim 8$ from the $b \to d\gamma$ process were derived for example in Ref. [136], to which we refer for details. With this, the scales derived in the text are set from

$$\delta C_{7\gamma}^{q} = \frac{16\pi^2}{V_{tb}^* V_{ts}} \frac{v}{m_b} \frac{v^2}{\Lambda^2} \mathcal{C}_{\gamma}^{3q} \;, \tag{A.11}$$

In the kaon sector, the operators are often written as

$$\mathcal{H}_{eff}^{EW} = C_{\gamma}^{\pm} Q_{\gamma}^{\pm} + h.c. \;, \;\; Q_{\gamma}^{\pm} = \frac{Q_d e}{16\pi^2} (\bar{s}_R \sigma_{\mu\nu} d_L \pm \bar{d}_R \sigma_{\mu\nu} s_L) F^{\mu\nu} \;, \tag{A.12}$$

with in the SM, $Q_d(C_{\gamma}^{+} \mp C_{\gamma}^{-}) = \sqrt{2} G_F m_{s(d)} V_{ts}^{\dagger} V_{td} D_0'(m_t/M_W)$, and $D_0'$ the Inami-Lim function [37]. These operators contribute to kaon observables, for example to $\varepsilon_K'$, $K \to \pi\pi\gamma$, or $K \to \pi\ell^+\ell^-$. However, contrary to the $b \to s\gamma$ process, it is very difficult to disentangle the genuine short-distance contribution from long-distance QED effects. As a result, the current sensitivity on the short distance is far from the SM level, with bounds in the range [59]

$$\frac{|\,{\rm Re}\, C_{\gamma}^{-}|}{G_F m_K} \lesssim 0.1 \;, \;\; \frac{|\,{\rm Im}\, C_{\gamma}^{-}|}{G_F m_K} \lesssim 0.2 \; [K^+ \to \pi^+\pi^0\gamma] \;, \tag{A.13}$$

$$\frac{|\,{\rm Re}\, C_{\gamma}^{+}|}{G_F m_K} \lesssim 0.3 \;\; [K \to \gamma\gamma] \;, \;\; \frac{|\,{\rm Im}\, C_{\gamma}^{+}|}{G_F m_K} \lesssim 0.03 \;\; [K_L \to \pi^0 e^+ e^-] \;. \tag{A.14}$$

The bound from $K_L \to \pi^0 e^+ e^-$ is stronger, but also less reliable than the others because it strongly depends on the absence of interference with other type of NP contributions. To stay on the conservative side, the bounds in the text are set using that from $|\,{\rm Re}\, C_{\gamma}^{+}|$,

$$\frac{16\pi^2}{Q_d} \frac{v}{\Lambda^2} \mathcal{C}_{\gamma}^{21} \lesssim 0.3 \times G_F m_K \;. \tag{A.15}$$

For comparison, we can now repeat the exercise with the leptonic magnetic operators, which are forbidden in the SM. For example, starting with

$$\mathcal{H}_{eff}^{NP} = \frac{e\mathcal{C}_{\gamma}^{IJ}}{\Lambda^2} \left(E^I \sigma_{\mu\nu} L^J\right) F^{\mu\nu} H^C + h.c. \tag{A.16}$$

the $\ell^I(P) \to \ell^J(p)\,\gamma(q)$ amplitude and rate are [40]

$$\mathcal{M}\left(\ell^I \to \ell^J \gamma\right) = ev \frac{\mathcal{C}_{\gamma}^{IJ}}{\Lambda^2} \{\bar{v}_{\ell^I}(P) P_L \sigma_{\mu\nu} v_{\ell^J}(p)\} q^{\nu} \varepsilon^{*\mu}(q) \;\to\; \Gamma\left(\ell^I \to \ell^J \gamma\right) = \frac{\alpha m_{\ell I}^3}{4\Lambda^2} \frac{v^2}{\Lambda^2} |\mathcal{C}_{\gamma}^{IJ}|^2 \;. \tag{A.17}$$

### EDM and flavor-blind magnetic penguins

Consider now the flavor-blind EDM operators, which we generalize to

$$\mathcal{H}_{eff} = \frac{e}{\Lambda^2} \mathcal{C}_u^{IJ} U^I \sigma_{\mu\nu} Q^J F^{\mu\nu} H + \frac{e}{\Lambda^2} \mathcal{C}_d^{IJ} D^I \sigma_{\mu\nu} Q^J F^{\mu\nu} H^C + \frac{e}{\Lambda^2} \mathcal{C}_e^{IJ} E^I \sigma_{\mu\nu} L^J F^{\mu\nu} H^C + h.c. \tag{A.18}$$

From them, we find for example

$$\mathcal{H}_{eff} \ni \frac{ev}{\Lambda^2} [\mathcal{C}_u^{II} \bar{\psi}_R^I \sigma_{\mu\nu} \psi_L^I + \mathcal{C}_u^{*II} \bar{\psi}_L^I \sigma_{\mu\nu} \psi u_R^I] F^{\mu\nu} \equiv e \frac{a_u}{4m_u} \bar{u} \sigma_{\mu\nu} u F^{\mu\nu} - \frac{i}{2} d_u \bar{u} \sigma_{\mu\nu} \gamma_5 u F^{\mu\nu} \;, \tag{A.19}$$

so,

$$a_{\psi^I} = \frac{4 v m_{\psi^I}}{\Lambda^2} {\rm Re}\, \mathcal{C}_{\psi}^{II} \;, \;\; \frac{d_{\psi^I}}{e} = \frac{2v}{\Lambda^2} {\rm Im}\, \mathcal{C}_{\psi}^{II} \;. \tag{A.20}$$

The EDM of the neutron is directly related to that of its constituent quarks though the precise proportionality factor is not entirely known, so we use the approximation $d_n \approx (4d_d - d_u)/3$ to derive the bounds given in the main text. Concerning $a_\mu$, requiring that it does not deviate by more than $3\sigma$ from the current best experimental result [137] $a_\mu \times 10^{10} = 11659208.0 \pm 6.3$ gives $\Lambda \gtrsim 350$ TeV. Note though that at present, theory is about $3\sigma$ away from experiment. It is difficult to translate this into a definitive NP scale though, because the theory prediction suffers from some rather uncertain hadronic contributions [138].

There is also the constraint on the EDM of the Hg nucleus, $d_{Hg}$, sensitive mainly on the $\mathcal{CP}$-violation occurring in the $\pi NN$ couplings. It can be induced by the chromomagnetic operators (among others)

$$\mathcal{H}_{eff} = \frac{g_s}{\Lambda^2}\tilde{\mathcal{C}}_u^{IJ} U^I \sigma_{\mu\nu} T^a Q^J G_a^{\mu\nu} H + \frac{g_s}{\Lambda^2}\tilde{\mathcal{C}}_d^{IJ} D^I \sigma_{\mu\nu} T^a Q^J G_a^{\mu\nu} H^C \; . \tag{A.21}$$

Phenomenologically, the EDM of Hg is expressed from the chromo-EDM of the quarks as [11]

$$\frac{d_{Hg}}{e} \approx 7 \times 10^{-3} \times (\tilde{d}_u - \tilde{d}_d) \; , \quad \tilde{d}_\psi = \frac{2v}{\Lambda^2} \operatorname{Im} \tilde{\mathcal{C}}_\psi \; . \tag{A.22}$$

The bounds in the text assume the electro- and chromomagnetic operators arise at the same scale, along with similar suppression mechanisms for $\mathcal{C}_\psi^{II}$ and $\tilde{\mathcal{C}}_\psi^{IJ}$.

## A.2 Complete list of six-fermion $\mathcal{B}$ and $\mathcal{L}$ violating operators

The full set of six-fermion operators can be organized into four classes:

$$\mathcal{H}_{eff} = \mathcal{H}_{|\Delta\mathcal{L}|=6} + \mathcal{H}_{\Delta\mathcal{L}=-3\Delta\mathcal{B}} + \mathcal{H}_{\Delta\mathcal{L}=+3\Delta\mathcal{B}} + \mathcal{H}_{|\Delta\mathcal{B}|=2} \; , \tag{A.23}$$

each with a finite number of vertices:

$$\mathcal{H}_{|\Delta\mathcal{L}|=6} = \frac{H^6}{\Lambda^{11}} L^6 + h.c. \; , \tag{A.24a}$$

$$\begin{aligned}\mathcal{H}_{\Delta\mathcal{L}=-3\Delta\mathcal{B}} = & \frac{H}{\Lambda^6} L^3 D^3 + \frac{H^3}{\Lambda^8}(L^3 U D^2 + L^3 Q^{\dagger 2} D + E^\dagger L^2 Q^\dagger D^2) \\ & + \frac{H^5}{\Lambda^{10}}(L^3 Q^{\dagger 2} U + E^\dagger L^2 Q^{\dagger 3}) + h.c. \; ,\end{aligned} \tag{A.24b}$$

$$\begin{aligned}\mathcal{H}_{\Delta\mathcal{L}=3\Delta\mathcal{B}} = & \frac{1}{\Lambda^5}(E L^{\dagger 2} U^3 + L^{\dagger 3} Q^\dagger U^2) \\ & + \frac{H^2}{\Lambda^7}(L^3 Q^3 + L^3 Q U^\dagger D^\dagger + E^\dagger L^2 D^\dagger U^{\dagger 2} + E^\dagger L^2 Q^2 U^\dagger + E^{\dagger 2} L Q U^{\dagger 2}) \\ & + \frac{H^4}{\Lambda^9}(L^3 Q D^{\dagger 2} + E^\dagger L^2 Q^2 D^\dagger + E^{\dagger 2} L Q^3) + h.c. \; ,\end{aligned} \tag{A.24c}$$

$$\begin{aligned}\mathcal{H}_{|\Delta\mathcal{B}|=2} = & \frac{1}{\Lambda^5}(U^2 D^4 + Q^{\dagger 2} U D^3 + Q^{\dagger 4} D^2) \\ & + \frac{H^2}{\Lambda^7}(Q^{\dagger 6} + Q^{\dagger 4} U D + Q^{\dagger 2} U^2 D^2 + Q^2 D^{\dagger 4}) + \frac{H^4}{\Lambda^9} Q^{\dagger 4} U^2 + h.c. \; .\end{aligned} \tag{A.24d}$$

For simplicity, the flavor contractions as well as the various possible spurion insertions are not written explicitly but can easily be constructed. Numerical coefficients are also understood, while $\Lambda$ represents the typical energy scale of the process generating these non-renormalizable interactions.

These series of interactions terminate because of the $SU(2)_L$ contraction $H^a H_a = 0$. Color and spinor contractions are understood. All the above interactions involve an even number of hermitian conjugated fields. In the two component notation, $X$ ($X^\dagger$) with $X = Q, U, D, L, E$ has an undotted (dotted) index. Dotted and undotted indices are either contracted together or pairs of dotted-undotted indices are contracted with $\sigma^\mu_{\alpha\dot{\alpha}}$ (which is then reducible using Fierz identities). Interactions involving gauge fields are possible, but are not written explicitly since gauge boson cannot change the chiral structure of the fermion current. The operators with an odd number of dotted and undotted indices are also permitted, at the cost of both a Higgs field and a covariant derivative acting on the fields (which we denote generically by a prefactor $\delta$)

$$\delta\mathcal{H}_{\Delta\mathcal{L}=-3\Delta\mathcal{B}} = \frac{\delta H^2}{\Lambda^8}(E^\dagger L^2 D^3 + L^3 Q^\dagger D^2) + \frac{\delta H^4}{\Lambda^{10}}(L^3 Q^\dagger UD + E^\dagger L^2 Q^{\dagger 2} D + L^3 Q^{\dagger 3}) + h.c.\ , \quad \text{(A.25a)}$$

$$\delta\mathcal{H}_{\Delta\mathcal{L}=3\Delta\mathcal{B}} = \frac{\delta H}{\Lambda^7}(L^{\dagger 3}U^3 + L^3 D^\dagger U^{\dagger 2} + E^{\dagger 2} L U^{\dagger 3} + E^\dagger L^2 Q U^{\dagger 2} + L^3 Q^2 U^\dagger)$$
$$+ \frac{\delta H^3}{\Lambda^9}(L^3 U D^{\dagger 2} + E^\dagger L^2 Q U^\dagger D^\dagger + L^3 Q^2 D^\dagger + E^{\dagger 2} L Q^2 U^\dagger + E^\dagger L^2 Q^3) + h.c.\ , \quad \text{(A.25b)}$$

$$\delta\mathcal{H}_{|\Delta\mathcal{B}|=2} = \frac{\delta H}{\Lambda^7}(QU^\dagger D^{\dagger 4} + Q^\dagger U^2 D^3 + Q^3 D^{\dagger 3} + Q^{\dagger 3} U D^2 + Q^{\dagger 5} D)$$
$$+ \frac{\delta H^3}{\Lambda^9}(Q^{\dagger 3} U^2 D + Q^{\dagger 5} U) + h.c.\ . \quad \text{(A.25c)}$$

Interactions with more derivatives do not induce new flavor transitions since they involve the same fields as those of Eq. (A.24) when of $\mathcal{O}(\delta^{2n})$, or as those of Eq. (A.25) when of $\mathcal{O}(\delta^{2n+1})$.

Not all the interactions above require Yukawa spurions to be invariant under $SU(3)^5$, but whenever they do not, Higgs fields are present. For example, the $L^3Q^3$ structure is immediately invariant but require two Higgs field to be gauge invariant. In this respect, within each class, all the couplings can be seen as seeded from (at least) one of the simplest interactions, from which they derive by repetitively substituting $U \to H^* Q^\dagger/\Lambda$, $D \to HQ^\dagger/\Lambda$, $Q \to HD^\dagger/\Lambda$, $Q \to H^* U^\dagger/\Lambda$, $E \to HL^\dagger/\Lambda$, or $L \to HE^\dagger/\Lambda$. If this picture really reflects the underlying dynamics, these insertions would always be accompanied by Yukawa insertions. For example, $H^2L^3Q^3$ can be understood as originating from $L^3QU^{\dagger 2}$, with two Higgs fields emitted from the $U^\dagger$ quarks. By contrast, the Higgless interactions all necessitate some Yukawa insertions to be invariant under $SU(3)^5$.

## A.3 Cayley-Hamilton Theorem

The Cayley-Hamilton Theorem states that any $n \times n$ square matrix $\mathbf{X}$ is solution of its own characteristic equation, once extrapolated to matrix form

$$p(\lambda) = \det[\mathbf{X} - \lambda\mathbf{1}] \Rightarrow p(\mathbf{X}) = \mathbf{0}\ . \quad \text{(A.26)}$$

At first glance, one may think this is trivial. It is tempting to write $p(\mathbf{X}) = \det[\mathbf{X} - \mathbf{X}\cdot\mathbf{1}] = 0$, but this is not correct because $\mathbf{X} - \lambda\mathbf{1}$ makes sense only for $\lambda \in \Re$, and $\det[\mathbf{X} - \mathbf{X}\cdot\mathbf{1}]$ is a scalar while $p(\mathbf{X})$ should equate to the null matrix. The demonstration when $\mathbf{X}$ is diagonalizable is straightforward though. By definition, any eigenvalue $\lambda$ satisfies

$$p(\lambda) = \lambda^n + c_{n-1}\lambda^{n-1} + ... + c_0 1 = 0\ . \quad \text{(A.27)}$$

Besides, there exists a vector $\mathbf{v}$ such that $\mathbf{X}\cdot\mathbf{v} = \lambda\mathbf{v}$, which means that

$$\mathbf{X}^n\cdot\mathbf{v} + c_{n-1}\mathbf{X}^{n-1}\cdot\mathbf{v} + ... + c_0\mathbf{1}\cdot\mathbf{v} = p(\lambda)\,\mathbf{v}\ , \quad \text{(A.28)}$$

Since this is valid for all the eigenvalues of $\mathbf{X}$, we can deduce that $\mathbf{X}^n + c_{n-1}\mathbf{X}^{n-1} + ... + c_0\mathbf{1} = \mathbf{0}$, i.e., $p(\mathbf{X}) = \mathbf{0}$. The theorem remains valid for non-diagonalizable matrices, but the demonstration in the general case will not be detailed here.

Specializing to $3 \times 3$ hermitian matrices, the three eigenvalues $\lambda_{1,2,3}$ of $\mathbf{X}$ can be expressed back in terms of traces and determinant of $\mathbf{X}$, hence:

$$p\,(\mathbf{X}) = (\mathbf{X} - \lambda_1 \mathbf{1})\,(\mathbf{X} - \lambda_2 \mathbf{1})\,(\mathbf{X} - \lambda_3 \mathbf{1}) = \mathbf{X}^3 - \langle \mathbf{X} \rangle \mathbf{X}^2 + \frac{1}{2}\mathbf{X}(\langle \mathbf{X} \rangle^2 - \langle \mathbf{X}^2 \rangle) - \det \mathbf{X} = 0\ . \quad \text{(A.29)}$$

Taking the trace of this equation, $\det \mathbf{X}$ can be eliminated as

$$\det \mathbf{X} = \frac{1}{3}\langle \mathbf{X}^3 \rangle - \frac{1}{2}\langle \mathbf{X} \rangle \langle \mathbf{X}^2 \rangle + \frac{1}{6}\langle \mathbf{X} \rangle^3\ . \quad \text{(A.30)}$$

Additional identities can be derived by expressing $\mathbf{X} = x_1 \mathbf{X}_1 + x_2 \mathbf{X}_2 + ...$ and extracting a given power of $x_1$, $x_2$,.... For example, taking $\mathbf{X} = x\mathbf{X} + y\mathbf{Y}$, and extracting $x^2 y$:

$$\begin{aligned} &\mathbf{X}^2\mathbf{Y} + \mathbf{XYX} + \mathbf{YX}^2 - \langle \mathbf{X}^2\mathbf{Y} \rangle - \mathbf{X}^2 \langle \mathbf{Y} \rangle - \langle \mathbf{X} \rangle\,(\mathbf{XY} + \mathbf{YX} - \langle \mathbf{XY} \rangle) \\ &\quad = \mathbf{X}\,(\langle \mathbf{XY} \rangle - \langle \mathbf{X} \rangle \langle \mathbf{Y} \rangle) - \frac{1}{2}(\mathbf{Y} - \langle \mathbf{Y} \rangle)\left(\langle \mathbf{X} \rangle^2 - \langle \mathbf{X}^2 \rangle\right)\ . \end{aligned} \quad \text{(A.31)}$$

Combining the definition of the determinant, $\varepsilon^{LMN}\mathbf{X}^{LI}\mathbf{X}^{MJ}\mathbf{X}^{NK} \equiv \varepsilon^{IJK} \det \mathbf{X}$, with the Cayley-Hamilton theorem leads to several useful identities. The starting point is Eq. (A.30):

$$\varepsilon^{LMN}\mathbf{X}^{LI}\mathbf{X}^{MJ}\mathbf{X}^{NK} \equiv \varepsilon^{IJK} \det \mathbf{X} = \varepsilon^{IJK}[\ \tfrac{1}{3}\langle \mathbf{X}^3 \rangle - \tfrac{1}{2}\langle \mathbf{X} \rangle \langle \mathbf{X}^2 \rangle + \tfrac{1}{6}\langle \mathbf{X} \rangle^3\ ]\ . \quad \text{(A.32)}$$

We can derive simpler identities involving traces and antisymmetric contractions by shifting $\mathbf{X} \to \mathbf{1} + \mathbf{X}$, expand in $\mathbf{X}$, and extract terms linear and quadratic in $\mathbf{X}$:

$$\varepsilon^{LJK}\mathbf{X}^{LI} + \varepsilon^{ILK}\mathbf{X}^{LJ} + \varepsilon^{IJL}\mathbf{X}^{LK} = \varepsilon^{IJK}\langle \mathbf{X} \rangle\ , \quad \text{(A.33a)}$$

$$\varepsilon^{LMK}\mathbf{X}^{LI}\mathbf{X}^{MJ} + \varepsilon^{LJM}\mathbf{X}^{LI}\mathbf{X}^{MK} + \varepsilon^{ILM}\mathbf{X}^{LJ}\mathbf{X}^{MK} = \varepsilon^{IJK}\tfrac{1}{2}[\ \langle \mathbf{X} \rangle^2 - \langle \mathbf{X}^2 \rangle\ ]\ . \quad \text{(A.33b)}$$

Other useful identities are derived by multiplying the definition of the determinant by $\mathbf{X}^{-1}$,

$$(\mathbf{X}^{-1})^{PK}\varepsilon^{IJP} \det \mathbf{X} = \varepsilon^{LMN}\mathbf{X}^{LI}\mathbf{X}^{MJ}\mathbf{X}^{NP}(\mathbf{X}^{-1})^{PK} = \varepsilon^{LMK}\mathbf{X}^{LI}\mathbf{X}^{MJ}\ . \quad \text{(A.34)}$$

The left-hand side can be simplified using the Cayley-Hamilton theorem. Multiplying both sides of Eq. (A.29) by $\mathbf{X}^{-1}$ leads to

$$\varepsilon^{ILM}\mathbf{X}^{LJ}\mathbf{X}^{MK} = \varepsilon^{LJK}[\ \mathbf{X}^2 - \langle \mathbf{X} \rangle \mathbf{X} + \tfrac{1}{2}\langle \mathbf{X} \rangle^2 - \tfrac{1}{2}\langle \mathbf{X}^2 \rangle\ ]^{LI}\ . \quad \text{(A.35)}$$

Finally, there are also identities with several different matrices. For example, by plugging $\mathbf{X} \to \mathbf{X} + \mathbf{Y}$ in Eq. (A.35), we can derive

$$\varepsilon^{ILM}(\mathbf{X}^{LJ}\mathbf{Y}^{MK} + \mathbf{Y}^{LJ}\mathbf{X}^{MK}) = \varepsilon^{LJK}\left[\ \{\mathbf{X}, \mathbf{Y}\} - \langle \mathbf{X} \rangle \mathbf{Y} - \langle \mathbf{Y} \rangle \mathbf{X} + \langle \mathbf{X} \rangle \langle \mathbf{Y} \rangle - \langle \mathbf{XY} \rangle\ \right]^{LI}\ . \quad \text{(A.36)}$$

The most general three-matrix identity is found by replacing $\mathbf{X} \to \mathbf{X} + \mathbf{Y} + \mathbf{Z}$ in Eq. (A.32),

$$\varepsilon^{LMN}\{\mathbf{X}, \mathbf{Y}, \mathbf{Z}\}^{LI,MJ,NK} = \varepsilon^{IJK}\left[\langle \mathbf{XYZ} + \mathbf{ZYX} \rangle - \langle \mathbf{X} \rangle \langle \mathbf{ZY} \rangle - \langle \mathbf{Y} \rangle \langle \mathbf{XZ} \rangle - \langle \mathbf{Z} \rangle \langle \mathbf{XY} \rangle + \langle \mathbf{X} \rangle \langle \mathbf{Y} \rangle \langle \mathbf{Z} \rangle\right], \quad \text{(A.37)}$$

where $\{\mathbf{X}, \mathbf{Y}, \mathbf{Z}\}^{a,b,c} \equiv \mathbf{X}^a\mathbf{Y}^b\mathbf{Z}^c + \mathbf{X}^a\mathbf{Z}^b\mathbf{Y}^c + \mathbf{Y}^a\mathbf{X}^b\mathbf{Z}^c + \mathbf{Y}^a\mathbf{Z}^b\mathbf{X}^c + \mathbf{Z}^a\mathbf{X}^b\mathbf{Y}^c + \mathbf{Z}^a\mathbf{Y}^b\mathbf{X}^c$. From this, simpler identities can be obtained by setting some matrices to $\mathbf{1}$ and/or equating some of them. For example, when $\mathbf{Z} = \mathbf{X}$, Eq. (A.37) reduces to

$$\begin{aligned} &\varepsilon^{LMN}(\mathbf{X}^{LI}\mathbf{X}^{MJ}\mathbf{Y}^{NK} + \mathbf{X}^{LI}\mathbf{Y}^{MJ}\mathbf{X}^{NK} + \mathbf{Y}^{LI}\mathbf{X}^{MJ}\mathbf{X}^{NK}) \\ &\qquad = \varepsilon^{IJK}\left[\ \langle \mathbf{X}^2\mathbf{Y} \rangle - \langle \mathbf{X} \rangle \langle \mathbf{YX} \rangle + \tfrac{1}{2}\langle \mathbf{Y} \rangle (\langle \mathbf{X} \rangle^2 - \langle \mathbf{X}^2 \rangle)\ \right]\ . \end{aligned} \quad \text{(A.38)}$$

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